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MAY 2016 **R/P/ID 37478/PMANH**

Time : Three hours Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define a Banach algebra.

2. Define an algebra.

3. State Tychonoff's theorem.

4. Define Hausdorff space.

5. Define totally disconnected space.

6. Define a locally connected space.

7. State closed graph theorem.

8. Define Hilbert space.

9. Define topological divisors of zero.

10. Define B^*-algebra.

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PART B — (5 × 7 = 35 marks)

Answer any FIVE questions.

11. Let X and Y be metric spaces and f be a mapping of X into Y. Prove that f is continuous if and only if $x_n \to x \Rightarrow f(x_n) \to f(x)$.

12. Prove that a topological space is compact if and only if every basic open cover has a finite subcover.

13. Prove that any continuous mapping of a compact metric space into a metric space is uniformly continuous.

14. Prove that a topological space X is disconnected $\Leftrightarrow$ there exists a continuous mapping of X onto the discrete two point space $\{0,1\}$.

15. Prove that the mapping $x \to F_x$ is an isometric isomorphism of N into N^{**}.

16. State and prove schwarz inequality.

17. Prove that every element x for which $\|x-1\|<1$ is regular, and the inverse of such and element is given by $x^{-1} = 1 + \sum_{n=1}^{\infty} (1-x)^n$.

18. If f_1 and f_2 are functionals on a Banach algebra A with the same null space M, prove that $f_1 = f_2$.

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PART C — (3 × 15 = 45 marks)

Answer any THREE questions.

19. Let X be a non-empty set and let there be given *closure* operation which assign to each subset of X a subset $\overline{A}$ of X in such a manner that

(a) $\overline{\emptyset} = \emptyset$

(b) $A \subseteq \overline{A}$

(c) $\overline{\overline{A}} = A$

(d) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

If a *closed* set A is defined to be one for which $A = \overline{A}$, prove that the class of all complements of such sets is a topology on X, whose operation is a precisely that initially given.

20. State and prove Ascoli's theorem.

21. State and prove the Weierstrass approximation theorem.

22. State and prove the closed graph theorem.

23. Prove that $r(x) = \lim \|x\|^{1/n}$.

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