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DECEMBER 2015 **P/ID 37457/PMAH**

Time : Three hours Maximum : 100 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define a normed linear space.

2. Define metrizable space.

3. State Heine Borel theorem.

4. Define sequentially compact metric space.

5. Define a connected space.

6. Define a locally connected space.

7. State closed graph theorem.

8. Define Hilbert space.

9. Define Banach algebra.

10. Define B^* algebra.

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SECTION B — (5 × 6 = 30 marks)

Answer ALL questions.

11. (a) State and prove Cantor's intersection theorem.

Or

(b) State and prove Cauchy's inequality.

12. (a) Prove that any continuous image of a compact space is compact.

Or

(b) State and prove Tychonoff's theorem.

13. (a) Prove that a topological space X is disconnected $\Leftrightarrow$ there exists a continuous mapping of X onto the discrete two point space {0, 1}.

Or

(b) Prove that the mapping $x \to F_x$ is an isometric isomorphism of N into N^{**}.

14. (a) If M is a proper closed linear subspace of a Hilbert space H, prove that there exists a non-zero vector z_0 in H such that $z_0 \perp M$.

Or

(b) If T is an operator on H for which $(Tx, x) = 0$ for all x, prove that $T = 0$.

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15. (a) Prove that every element x for which $\|x-1\|<1$ is regular, and the inverse of such an element is given by $x^{-1}=1+\sum_{n=1}^{\infty}(1-x)^n$.

Or

(b) If I is a proper closed two sided ideal in A, prove that the quotient algebra A/I is a Banach algebra.

SECTION C — (5 × 10 = 50 marks)

Answer ALL questions.

16. (a) Let X be a non-empty set and let there be given closure operation which assign to each subset of X a subset $\overline{A}$ of X in such a manner that

(i) $\overline{\phi}=\phi$

(ii) $A\subseteq\overline{A}$

(iii) $\overline{\overline{A}}=A$

(iv) $\overline{A\cup B}=\overline{A}\cup\overline{B}$

If a closed set A is defined to be one for which $A=\overline{A}$ prove that the class of all complements of such sets is a topology on X, whose operation is a precisely that initially given.

Or

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(b) (i) State and prove Lindelof's theorem.

(ii) Prove that every separable metric space is second countable.

17\. (a) State and prove Lebesgue's covering lemma.

Or

(b) State and prove Tietze extension theorem.

18\. (a) State and prove the Weierstrass approximation theorem.

Or

(b) State and prove Hahn-Banach theorem. (Prove the Lemma also).

19\. (a) State and prove the Open mapping theorem. (Prove the Lemma also).

Or

(b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

20\. (a) Prove that $\sigma(x)$ is non-empty.

Or

(b) State and prove Gelfand-Neumark representation theorem.

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