

(6 pages)

OCTOBER 2013

**P/ID 17456/RCF/
PCAB**

Time : Three hours

Maximum : 75 marks

PART A — (5 × 5 = 25 marks)

Answer ALL questions.

All questions carry equal marks.

1. (a) If $P(A \cup B) = 4/7$, $P(A \cap B) = 1/6$ and $P(\bar{A}) = 3/5$, find $P(A)$ and $P(B)$. Also, examine the independence of A and B .

Or

- (b) State the conditions satisfied by a probability density function. Verify whether the following function satisfies such conditions

$$f(x) = \begin{cases} 6(2-x)(x-1), & \text{if } 1 < x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

2. (a) Define Negative Binomial(5, 1/3) distribution. Also, find its mean.

Or

- (b) If a random variable X is distributed according to the Exponential distribution, compute the probabilities $P(X < 1)$ and $P(|X| < 3)$.

3. (a) Ten competitors in a contest were ranked by two judges as follows :

Judge 1 : 6 4 9 8 1 2 3 10 5 7

Judge 2 : 3 5 8 4 7 10 2 1 6 9

Calculate the coefficient of correlation between the rankings using Spearman's formula.

Or

- (b) Describe the method of fitting the equation $y = ab^x$.

4. (a) What is systematic random sample? Describe the method of drawing a systematic random sample.

Or

- (b) Find 95% confidence limits to the population proportion when the sample proportion is determined as 0.46 from a random sample of 95 observations.

5. (a) Define replication. Explain its relevance in scientific experiments.

Or

- (b) Describe short term fluctuations with a suitable example.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

All questions carry equal marks.

6. Three urns contain black and white balls as follows :

Urn-1 : 2 Black, 1 White

Urn-2 : 1 Black, 2 White

Urn-3 : 2 Black, 2 White

One of the urns is selected at random and one ball is drawn randomly. It is found that the ball drawn is white. What is the probability that the ball has been drawn from (i) Urn-2 (ii) Urn-3?

7. Determine the value of k so that the following function is a probability density function. Also, find the coefficient of variation of the respective distribution.

$$f(x) = \begin{cases} kx, & \text{if } 0 < x < 5 \\ k(10 - x), & \text{if } 5 \leq x < 10 \\ 0, & \text{otherwise} \end{cases}$$

8. The probability density function of the joint probability distribution of the random variables X and Y is given by

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$$f(x, y) = \begin{cases} 6(1 - x - y), & \text{if } x > 0, y > 0 \text{ and } x + y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the marginal distributions of X and Y . Also, find the conditional distribution of X given $Y = y$.

9. Obtain the moment generating function of the normal distribution. Also, examine its symmetric property.
10. The amount of money spent (Y , in Rs.100,000's) on Research and Development by a Corporation is assumed to depend upon its gross sales (X , in Rs. 10,00,000's). The information regarding X and Y for the past 9 years is given as follows:

Y: 1.9 3.4 6.5 5.7 2.8 2.3 6.2 7.6 5.0

X: 2.8 3.2 3.0 4.0 3.8 3.4 3.2 4.7 5.2

Fit a linear regression model of Y on X . Using the model, estimate the amount of money spent by the Corporation when its gross sales is Rs. 45, 00, 000.

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11. Test whether the means of the populations from which the following samples have been drawn are equal :

Sample 1 : 25 24 22 27 16 19 18 23 26

Sample 2 : 37 43 28 34 35 33 30 41 38 32 42

Assume that the populations are normal having equal variance and further assume that the samples are independent. -

12. A company is interested in determining whether there is an association between commuting time of their employees and the level of stress-related problems observed on the job. A study was conducted with 105 randomly selected employees and the results of the study are given in the following table :

Commuting time (in minutes)	Stress		
	High	Moderate	Low
Below 20	8	6	11
20–50	13	7	28
More than 50	15	4	13

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At 5% level of significance, test whether it is evident that there is a significant relationship between commuting time and the level of stress-related problems.

13. The following table shows the number of units of production per day turned out by four different types of machines during three shifts :

Shift	Machine			
	I	II	III	IV
1	40	36	45	30
2	38	42	50	41
3	36	30	48	35

Using analysis of variance, test at 5% level of significance whether (a) mean production of four machines are equal (b) mean production during the three shifts are equal.
