(6 pages)

OCTOBER 2012 **P/ID 40003/PPHC**

Time : Three hours Maximum : 100 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

All questions carry equal marks.

1. List the admissibility conditions of a wave function.

2. State uncertainty principle.

3. What are spherical harmonics?

4. Show that $\sin 2x$ is not an eigen function of the operator $\frac{d}{dx}$ but of $\frac{d^2}{dx^2}$.

5. Write down the time evolution operator.

6. Define inner products and outer products in terms of Dirac's bra and ket.

7. The second-order correction to the energy of the ground state is always negative. Why?

8. What is WKB quantization rule?

9. What are singlet and triplet states?

10. What are Clebsh-Gordan coefficients?

PART B — ($5 \times 6 = 30$ marks)

Answer ALL questions.

All questions carry equal marks.

11. (a) Describe the different postulates of quantum mechanics.

Or

(b) Derive the probability density P and current density J and hence the continuity equation.

12. (a) Separate the Schrödinger equation of a particle in spherically symmetric potential into three equations by separate variable method.

Or

(b) Write the Hamiltonian for hydrogen atom and reduce the two-body hydrogen problem into one-body problem.

13. (a) Discuss the effect of time reversal in the time-dependent Schrödinger equation.

Or

(b) Show that displacement in time leads to conservation of energy.

14. (a) Obtain the energy eigen values of harmonic oscillator by the WKB method.

Or

(b) State and prove the upper bound theorem of the variation method.

15. (a) Derive the matrices for the operators J^2, J_z, J_x and J_y for $j = 1/2$.

Or

(b) For Pauli's matrices, prove that :

(i) $\lfloor \sigma_x, \ \sigma_y \rfloor = 2i\sigma_z$

(ii) $\sigma_x \sigma_y \sigma_z = i$.

PART C — (5 × 10 = 50 marks)

Answer ALL questions.

All questions carry equal marks.

16. (a) Show that the commutator

$[x, [x, H]] = \frac{\eta^2}{m}$, where H is the Hamiltonian operator.

Or

(b) Illustrate the uncertainty principle on the basis of single-slit experiment. Derive the ground state energy of the hydrogen using uncertainty principle.

17. (a) Derive the energy eigen functions of a particle in a finite potential well and describe the method of finding the corresponding energy eigen values.

Or

(b) Solve the Schrödinger equation for the linear harmonic oscillator and obtain the energy eigen values and the corresponding energy eigen functions.

18. (a) Show that the total angular momentum of a particle with spin is the generators of the infinitesimal rotation.

Or

(b) Derive the equations of motion for states and operators in the Heisenberg and interaction pictures.

19. (a) Calculate the first-order correction to the ground state energy of an anharmonic oscillator of mass m and angular frequency ω subjected to a potential

$$V(x) = \frac{1}{2} m\omega^2 x^2 + bx^4$$

Or

(b) Estimate the ground state energy of the hydrogen atom using $e^{-\alpha r}$ as the trial function.

20. (a) Derive the energy eigen values and the corresponding energy eigen functions of the non-relativistic Hamiltonian including spin of an electron.

Or

(b) Obtain the Clebsh-Gordan coefficients for a system having $j_1 = 1$ and $j_2 = 1/2$.
