(6 pages)

MAY 2013 **P/ID 37454/PMAD**

Time : Three hours Maximum : 100 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

Each question carries 2 marks.

1. Define Borel field of events and random events.
2. Define moment of order k.
3. Define the generating function of a random variable.
4. Define Pòlya distribution.
5. State Poisson's law of large numbers.
6. Define χ^2 statistic.
7. Define parametric hypothesis and parametric test.
8. Define most efficient estimate.
9. Define null hypothesis and alternate hypothesis.
10. Define biased and unbiased tests.

SECTION B — (5 × 6 = 30 marks)

Answer ALL questions.

Each question carries 6 marks.

11. (a) Let $\{A_n\}$, $n = 1, 2, \ldots$, be a nonincreasing sequence of events and let A be their product. Prove that $P(A) = \lim_{n \to \infty} P(A_n)$.

Or

(b) Prove that the coefficient of correlation satisfies the double inequality

$$-1 \leq \rho \leq 1 .$$

12. (a) If the l th moment m_1 of a random variable exists, prove that it is expressed by

$$m_l = \frac{\phi^l(0)}{i^l},$$

where $\phi^l(0)$ is the l th derivative of the characteristic function $\phi(t)$ of this random variable at $t = 0$.

Or

(b) State and prove Poisson theorem.

13. (a) Let $F_n(x), (n = 1,2,...)$ be the distribution function of the random variable X_n. Prove that the sequence $\{X_n\}$ is stochastically convergent to zero if and only if the sequence $\{F_n(x)\}$ satisfies the relation

$$\lim_{n\to\infty} F_n(x) = \begin{cases} 0 & for \quad x \le 0 \\ 1 & for \quad x > 0 \end{cases}$$

Or

(b) Prove that the sequence $\{F_n(t)\}$ of distribution functions of Student's t with n degrees of freedom satisfies the relation

$$\lim_{n\to\infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-t^2} dt$$

14. (a) Explain independence tests by contingency tables.

Or

(b) The random variable X has the Poisson distribution

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, (k = 0,1,2,...)$$

where the parameter λ is unknown. Estimate λ by the method of maximum likelihood.

15. (a) The parameters m and σ of a characteristic x, which has a normal distribution $N(m, \sigma)$ are unknown. Test the hypothesis $H_0(m = 0, \sigma = 1)$ against the alternative $H_1(m = 1, \sigma = 1)$.

Or

(b) With the usual notation, prove that

$$E_{Q'}(n) \cong -\frac{\log A \log B}{E_{Q'}(z^2)}.$$

SECTION C — (5 × 10 = 50 marks)

Answer ALL the questions.

Each questions carry 10 marks.

16. (a) Let (X, Y) be a random variable of continuous type and let $f(x, y), f_1(x), f_2(y)$ denote respectively the densities of the random variables (X, Y), X and Y. Find the distribution of $X + Y$.

Or

(b) Prove that the equality $\rho^2 = 1$ is a necessary and sufficient condition for the relation $P(Y = aX + b) = 1$ to hold.

17. (a) Let $F(x)$ and $\phi(t)$ denote respectively the distribution function and the characteristic function of the random variable X. If $a + h$ and $a - h$ $(h > 0)$ are continuity points of the distribution function $F(x)$, prove that

$$F(a+h) - F(a-h) = \lim_{T \to \infty} \int_{-T}^{T} \frac{\sin ht}{t} e^{-ita} \phi(t) dt.$$

Or

(b) Find the density function of the random variable X whose characteristic function is

$$\phi(t) = \begin{cases} 1 - |t| & for \quad |t| \leq 1 \\ 0 & for \quad |t| > 1 \end{cases}$$

18. (a) State and prove de Moivre-Laplace theorem.

Or

(b) Derive the distribution of the statistic $(\overline{X}, S)$.

19. (a) State and prove Smirnov's theorem.

Or

(b) State and prove the Rao-Cramér inequality.

20. (a) Explain uniformly most powerful test.

Or

(b) Explain the sequential probability ratio test.

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