(6 pages)

OCTOBER 2013 **P/ID 37454/PMAD**

Time : Three hours Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

Each questions carries 2 marks.

1. Give the axiomatic definition of probability.
2. Define the marginal distribution function of a two dimensional random variable.
3. State Chebyshev's inequity.
4. What do you mean by regression hyper plane of second type?
5. Define characteristic function.
6. Define binomial distribution.
7. What do you mean by stochastically convergent to zero?
8. Define the term statistical hypothesis.
9. Define consistent estimate.
10. Sate Wald's fundamental identity.

PART B — (5 × 6 = 30 marks)

Answer ALL questions.

Each questions carries 6 makes.

11. (a) Let $\{A_n\}, n = 1, 2, ...$ be a non-decreasing sequence of events and let A be their alternative. Then prove that $P(A) = \lim_{n\to\infty} P(A_n)$.

Or

(b) Prove that if the events A and B are independent, then the same is true for the events $\overline{A}$ and $\overline{B}$.

12. (a) Find the mean and variance of Poisson distribution.

Or

(b) The random variable X takes the vales -1 and $+1$ with probabilities 0.5, 0.5. Find the characteristic function and hence find its mean.

13. (a) Prove that the sequence of random variables $\{X_n\}$ where $(X_n = Y_n - p, Y_n$ follows the following distribution

$$P\left(y_n = \frac{r}{n}\right) = \binom{n}{r} p^r (1-p)^{n-r}, \text{ where } 0 < p < 1,$$

$r = 0, 1, 2, \ldots.., n$ is stochastically convergent.

Or

(b) Show that the mean $\overline{X}$ and the variance S^2 of simple samples drawn from a population in which the characteristic X has a symmetric distribution are uncorrelated.

14. (a) We have good and defective items in a lot and the proportion p of defective items is unknown Test the hypothesis that $H_0 : p = 0.1$.

Or

(b) Consider the class D of distribution functions of random variable X whose second moment exists. Let the variance σ^2 of X be unknown. Estimate σ^2.

15. (a) Explain power function and OC function.

Or

(b) Prove that if the variance of Z is different from zero, the probability that $n = \infty$ is 0.

PART C — ($5 \times 10 = 50$ marks)

Answer ALL questions.

Each question carries 10 marks.

16. (a) Give an example of three dependent random variables X, Y, Z which are pairwise independent.

Or

(b) The distribution of (X, Y) is

$P(X = 1, Y = 1) = P(X = 1, Y = 2) =$

$P(X = 2, Y = 2) = \frac{1}{3}$. Find $F(x, y), F_1(x)$ and $F_2(x)$, the distribution functions.

17. (a) The joint probability distribution of (X, Y) is

$$f(x, y) = \begin{cases} \frac{1}{2}[1 + xy(x^2 - y^2)] & \text{for } |x| \le 1 \text{ and } |y| \le 1 \\ 0 & \text{for all other points} \end{cases}$$

Test whether X and Y are independent.

Or

(b) Find the mean of Normal distribution.

18. (a) The random variables $X_i (i = 1, 2, \ldots)$ are independent and have the some probability distribution,

$$P(X_i = 0) = P(X_i = 3) = P(X_i = 7) = P(X_i = 12) = \frac{1}{4}.$$

Check whether for this sequence the local limit theorem of Gnedenko holds.

Or

(b) Find the distribution of mean of independent normally distributed random variables.

19. (a) The following sample correlation coefficients were obtained from a simple sample of size 7 drawn from a three-dimensional normal population :

$r_{12} = 0.2$, $r_{13} = 0.4$, $r_{23} = -0.35$. Test the hypothesis H_0 $(\rho_{12} = \rho_{13} = \rho_{23} = 0)$ at the significance level 0.05.

Or

(b) State and prove Rao-Cramer inequality.

20. (a) Write a note on multiple classification.

Or

(b) Represent the sequential probability ratio test procedure in the form of a Markov chain.

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