MAY 2016 **P/ID 37474/PMAND**

Time : Three hours Maximum : 100 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. State Baye's theorem.

2. Define absolute moment of order k.

3. Define one-point distribution.

4. Define Cauchy distribution.

5. Write Kolmogorov inequality.

6. State Prohorov theorem.

7. State Kolmogorov theorem.

8. Define unbiased estimate.

9. Define composite hypothesis.

10. Define sequential analysis.

SECTION B — (5 × 7 = 35 marks)

Answer any FIVE questions.

11. Let (X, Y) be a random variable of continuous type and let $f(x, y)$, $f_1(x)$, $f_2(y)$ denote respectively the densities of the random variables (X, Y), X and Y. Find the distribution of $X + Y$.

12. Derive Lapunov inequality.

13. If the l th moment m_l of a random variable exists, prove that it is expressed by $m_l = \frac{\phi^l(0)}{i^l}$,

    where $\phi^l(0)$ is the l-th derivative of the characteristic function $\phi(t)$ of this random variable at $t = 0$.

14. The random variable X has the distribution $N(1; 2)$. Find the probability that X is greater than 3 in absolute value.

15. State and prove Borel-Cantelli Lemma.

16. Suppose that the characteristic X of elements of a population has the normal distribution $N(m; \sigma)$ with m unknown. The median of the simple sample of size n is taken as an estimate of m. Find the asymptotic efficiency of this estimate.

17. The expected value m of a population, where the characteristic x has the normal distribution $N(m;1)$, with m unknown. Test the hypothesis $H_0(m = m_0)$ against the alternate hypothesis $H_1(m = m_1)$. Find a most powerful test for testing this hypothesis based on a simple sample.

18. State and prove Fisher's Lemma.

SECTION C — (3 × 15 = 45 marks)

Answer any THREE questions.

19. The random variables X and Y have the joint density given by

$$f(x,y) = \frac{1}{2\pi}\exp\left(-\frac{x^2 - 2xy + 2y^2}{2}\right).$$

Compute the moments of first and second order. Also find the correlation coefficient.

20. Let $F(x)$ and $\phi(t)$ denote respectively the distribution function and the characteristic function of the random variable X. If $a+h$ and $a-h\,(h>0)$ are continuity points of the distribution function $F(x)$, prove that

$$F(a+h) - F(a-h) = \lim_{T\to\infty}\int_{-T}^{T}\frac{\sin ht}{t}e^{-ita}\phi(t)\,dt\,.$$

21. State and prove de Moivre Laplace theorem.

22. Explain independence tests by contingency tables.

23. State and prove Wald's fundamental identity.

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