(6 pages)

MAY 2016 **P/ID 37454/PMAD**

Time : Three hours Maximum : 100 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Compute the probability that heads appear atleast twice in three consecutive tosses of a coin.

2. The random variable X can take two values 2 and 4, where $P(X = 2) = 0.2$ and $P(X = 4) = 0.8$. Find. $E(X^2)$

3. Write any two properties of characteristic function.

4. Define the probability generating function.

5. State Chebyshev's law of large numbers.

6. Define simple random sample.

7. Define non-parametric hypothesis and non-parametric tests.

8. Define the likelihood function of the purely continuous Markov process.

9. Define most powerful test.

10. What is sequential analysis?

SECTION B — (5 × 6 = 30 marks)

Answer ALL questions.

11. (a) Find the mean and variance of the binomial distribution.

Or

(b) Prove that the expected value of the product of an arbitrary finite number of independent random variables, whose expected values exist, equals the product of the expected values of these variables.

12. (a) Compute the semi-invariants of the Poisson distribution, given by

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}.$$

Or

(b) State and prove Poisson's theorem.

13. (a) Let $F_n(x)$, $(n = 1, 2, ...)$ be the distribution function of the random variable X_n. Prove that the sequence $\{X_n\}$ is stochastically convergent to zero if and only if the sequence $\{F_n(x)\}$ satisfies the relation

$$\lim_{n\to\infty} F_n(x) = \begin{cases} 0 & \text{for } x \le 0 \\ 1 & \text{for } x > 0 \end{cases}.$$

Or

(b) From a population in which the characteristic X has the same normal distribution $N(1, 2)$, we draw a simple sample of size $n = 12$. Observe the following values of X:

$x_1 = 2.0$ $x_2 = 2.5$ $x_3 = 0.5$ $x_4 = 1.0$

$x_5 = 0.0$ $x_6 = -0.9$ $x_7 = .15$ $x_8 = -1.5$

$x_9 = 0.8$ $x_{10} = 1.1$ $x_{11} = 0.8$ $x_{12} = 0.4$.

Find $\bar{x}$ and s^2. What is the probability that Z will exceed or equal to 32.28?

14. (a) Suppose in a simple sample $n = 150$ elements. The mean and the standard deviation of the sample are, respectively, $\bar{x} = 0.4$ and $s = 4$. The expected value and the standard deviation of the population are not known. Should we reject the hypothesis $H_0(m = 0)$?

Or

(b) The random variable X has the Poisson distribution

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, \ (k = 0, 1, 2, ...)$$

where the parameter λ is unknown. Estimate λ by the method of maximum likelihood.

15. (a) Find a most powerful unbiased test for testing from simple samples, the hypothesis $H_0(m = m_0)$ against the alternative $H_1(m = m_1)$ where m is the unknown expected value of a Normal $N(m, 1)$ population and m_1 is an arbitrary number different from m_0.

Or

(b) Explain sequential probability ratio test.

SECTION C — (5 × 10 = 50 marks)

Answer ALL questions.

16. (a) State and prove Lapunov inequality.

Or

(b) The random variables X and Y have the joint density given by

$$f(x, y) = \frac{1}{2\pi} \exp\left(- \frac{x^2 - 2xy + 2y^2}{2}\right)$$

Compute the moments of first and second order. Also find the correlation coefficient.

17. (a) Let $F(x)$ and $\phi(t)$ denote respectively the distribution function and the characteristic function of the random variable X. If $a+h$ and $a-h\,(h>0)$ are continuity points of the distribution function $F(x)$, prove that

$$F(a+h)-F(a-h)=\lim_{T\to\infty}\int_{-T}^{T}\frac{\sin ht}{t}\,e^{-ita}\,\phi(t)\,dt\,.$$

Or

(b) Define Cauchy distribution and find its characteristic function.

18. (a) State and prove Lindberg-Levy theorem.

Or

(b) State and prove Kolmogorov theorem.

19. (a) Explain independence tests by contingency tables.

Or

(b) State and prove Blackwell's theorem.

20. (a) State and prove Fisher's lemma.

Or

(b) If $E(z) \neq 0$, prove that there exists one and only one real number $h_0 \neq 0$ such that $E(e^{h_0 z}) = 1$. Also, if $E(z) = 0$, prove that the above equality holds only for $h_0 = 0$.

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