

(7 pages)

DECEMBER 2015

P/ID 17414/RBR

Time : Three hours

Maximum : 75 marks

PART A — (5 × 5 = 25 marks)

Answer ALL questions.

1. (a) Solve graphically the following L.P.P.

$$\text{Maximize : } Z = 5x_1 + 3x_2$$

$$\text{Subject to constraints : } 3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0.$$

Or

- (b) Explain Big-M method.

2. (a) (i) Define dual problem.

- (ii) Write the dual to the following L.P.P.

$$\text{Maximize } Z = x_1 - x_2 + 3x_3$$

Subject to the constraints :

$$x_1 + x_2 + x_3 \leq 10$$

$$2x_1 - x_3 \leq 2$$

$$2x_1 + 3x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0.$$

Or

- (b) Explain least cost method of transportation problem.

3. (a) Write the dynamic programming algorithm.

Or

- (b) Use branch and bound method to solve the following linear programming problem :

$$\text{Maximize : } z = x_1 + 4x_2$$

Subject to the constraints :

$$2x_1 + 4x_2 \leq 7,$$

$$5x_1 + 3x_2 \leq 15,$$

$x_1, x_2 \geq 0$ and are integers.

4. (a) Draw the network diagram of the project

Activity : 1 – 2 1 – 3 2 – 3 2 – 4 3 – 4 4 – 5

Or

- (b) We have five jobs, each of which must go through the two Machines A and B in the order AB, processing times in hours are given in the table below :

Job (i) :	1	2	3	4	5
Machine A (A_j) :	5	1	9	3	10
Machine B (B_j) :	2	6	7	8	4

Determine a sequence for the five jobs that will minimize the total elapsed time.

5. (a) Two manufactures, X and Y are competing with each other in a very restricted market. The state – transition matrix for the market summarizes the probabilities that customers will move from one manufacturer to the other in any one month. Interpret the state – transition matrix in terms of (i) retention and loss and (ii) retention and gain

	From	To
	X	Y
X	0.7	0.3
Y	0.1	0.9

Or

- (b) A T.V. repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs set in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 – hour day, what is repairman’s expected idle time each day? How many jobs are head of the average set just brought in?

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

6. Use revised simplex method to solve the linear programming problem :

$$\text{Maximize : } Z = 5x_1 + 3x_2$$

Subject to constraints :

$$4x_1 + 5x_2 \geq 10,$$

$$5x_1 + 2x_2 \leq 10,$$

$$3x_1 + 8x_2 \leq 12,$$

$$x_1, x_2 \geq 0.$$

7. Use two – phase simplex method to

$$\text{Maximize } Z = 2x_1 + 3x_2 + 5x_3$$

Subject to the constraints :

$$3x_1 + 10x_2 + 5x_3 \leq 15,$$

$$33x_1 - 10x_2 + 9x_3 \leq 33,$$

$$x_1 + 2x_2 + x_3 \geq 4,$$

$$x_1, x_2, x_3 \geq 0.$$

8. Consider a transportation problem with $m = 3$ and $n = 4$ where

$$c_{11} = 2 \quad c_{12} = 3 \quad c_{13} = 11 \quad c_{14} = 7$$

$$c_{21} = 1 \quad c_{22} = 0 \quad c_{23} = 6 \quad c_{24} = 1$$

$$c_{31} = 5 \quad c_{32} = 8 \quad c_{33} = 15 \quad c_{34} = 9$$

Suppose $S_1 = 6$, $S_2 = 1$ and $S_3 = 10$, where as $D_1 = 7$, $D_2 = 5$, $D_3 = 3$ and $D_4 = 2$. Apply the transportation simplex method to find an optimal solution.

9. Using the bounded variable technique, solve the following linear programming problem :

$$\text{Maximize : } Z = x_2 + 3x_3$$

Subject to the constrains :

$$x_1 + x_2 + x_3 \leq 10,$$

$$x_1 - 2x_3 \geq 0 ;$$

$$2x_2 - x_3 \leq 10 ;$$

$$0 \leq x_1 \leq 8,$$

$$0 \leq x_2 \leq 4,$$

$$x_3 \geq 0.$$

10. Find the optimum integer solution to the L.P.P

$$\text{Maximize } Z = 3x_1 + 4x_2$$

Subject to the constraints :

$$3x_1 + 2x_2 \leq 8 ;$$

$$x_1 + 4x_2 \leq 10 ;$$

$x_1, x_2 \geq 0$ and are integers.

11. Solve the following integer programming problem :

$$\text{Maximize } Z = x_1 + 4x_2$$

Subject to the constraints :

$$2x_1 + 4x_2 \leq 7,$$

$$5x_1 + 3x_2 \leq 15,$$

$x_1, x_2 \geq 0$ are integers.

12. (a) A book binder has one printing press, one binding machine and the manuscripts of a number of different books. The times required to perform the printing and binding operations for each book are known. Determine the order in which the books should be processed in order to minimize the total time required to process all the books. Find also the total time required.

Processing time in minutes

Book :	1	2	3	4	5
Printing time :	40	90	80	60	50
Binding time :	50	60	20	30	40

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- (b) Suppose that an additional operation is added to the process described in (a) – viz. finishing. The times required for operations are given below.

	Finishing time in minutes				
Book :	1	2	3	4	5
Finishing time :	80	100	60	70	110

What is the order in which the books should be processed? Find also the minimal total elapsed time.

13. (a) A drive – in bank window has a mean service time of 2 minutes, while the customers arrive at a rate of 20 per hour. Assuming that these represent rates with a poisson distribution. Determine.
- (i) The proportion the teller will be idle.
 - (ii) How long a customer will wait before reaching the server?
 - (iii) What fraction of customers will have to wait in line?
 - (iv) The probability that a customer has to wait?
- (b) Explain queuing system.