(6 pages)

DECEMBER 2014 **P/ID 4519/XDG**

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 20 = 80 marks)

Answer ALL questions.

1. (a) (i) Define integer programming problem. (5)

(ii) Develop the Branch and bound tree for the following problem. For convenience, always select x_1 as the branching variable at node 0.

Maximize $Z = 2x_1 + 3x_2$

Subject to $5x_1 + 7x_2 \le 35$, $4x_1 + 9x_2 \le 36$, $x_1, x_2 \ge 0$ and integer. (15)

Or

(b) (i) Describe cutting plane algorithm. (5)

(ii) Solve the following problem by branch and bound method : (15)

Maximize $Z = 18x_1 + 14x_2 + 8x_3 + 4x_4$

Subject to

$15x_1 + 12x_2 + 7x_3 + 4x_4 + x_5 \le 37$,

$x_1, x_2, x_3, x_4, x_5 = (0, 1)$.

2. (a) (i) Examine the function $f(x) = x^4 + x^2$ for extreme points. (5)

(ii) Solve the following as a separate convex programming problem.

Minimize $Z = (x_1 - 2)^2 + 4(x_2 - 6)^2$

Subject to

$6x_1 + 3(x_2 + 1)^2 \leq 12, \quad x_1, x_2 \geq 0$. (15)

Or

(b) (i) Write the Kuhn-Tucker necessary conditions for the following problem : (5)

Minimize $f(x) = x_1{}^4 + x_2{}^2 + 5x_1x_2x_3$

Subject to $x_1{}^2 - x_2{}^2 + x_3{}^3 \leq 10$,

$x_1^3 + x_2^2 + 4x_3^2 \geq 20$

(ii) Consider the problem : (15)

Minimize $Z = 2x_1{}^2 + 2x_2{}^2 + 3x_3{}^2 + 2x_1x_2 + 2x_2x_3 + x_1 - 3x_2 - 5x_3$

Subject to $x_1 + x_2 + x_3 \geq 1$, $3x_1 + 2x_2 + x_3 \leq 6$, $x_1, x_2, x_3 \geq 0$. Show that Z is strictly convex and then solve by quadratic programming algorithm.

3. (a) (i) An item sells for \$ 25 a unit, but a 10% discount is offered for lots of 150 units or more. A company uses this item at the rate of 20 units per day. The set-up cost for ordering a lot is \$ 50 and the holding cost per unit per day is \$ 0.30. Should the company take advantage of the discount? (5)

(ii) The pdf of the demand per period in an infinite-horizon inventory model is given as $f(D) = .08D, \quad 0 \le D \le 5$. The unit cost parameters are selling price = \$10, purchase price = \$8, penalty cost = \$1, discount factor = .9. Determine the optimal inventory policy assuming zero delivery lag and that the unfilled demand is backlogged. (15)

Or

(b) (i) In his Engineering economy class, Professor Porter Stone teaches his students to "play" the stock market. The stock market game losts for 10 days and starts by assuming that the value of the selected stock will rise by 1% a day. In any one day, there is also a chance that the market will decline as given by the following table :

Day :	1	2	3	4	5	6	7	8	9	10
Percent decline :	5.	5.	.8	.1	1.4	.6	1.2	.2	.3	.1
P (Decline) :	.06	.02	.05	.07	.10	.13	.15	.20	.12	.10

The objective is to maximize the accumulated value of the stock. If you were a participant in Professor Stone's class, when should you start the game? (5)

(ii) A company is introducing a new product into the market. If the sales are high, there is a .5 probability that they will remain so next month. If they are not, the probability that they will become next month is only .2. If the company advertises and the sales are high, the probability that they will remain high next month will increase to .8. Conversely, an advertising campaign while the sales are low will raise the probability to only .4.

If no advertisement is used and the sales are high, the returns are expected to be 10 if the sales remain high next month and 4 if they do not. The corresponding returns if the product starts with high sales are 7 and –2.

Using advertisement will result in returns of 7 if the product starts with high sale and continues to be so and 6 if it does not. If the sales start low, the returns are 3 and –5, depending on whether or not they remain high. Determine the company's optimal policy over the next 3 months. (15)

4. (a) (i) Prove that the mean and standard deviation of exponential distribution are equal. (5)

(ii) Discuss (M/M/C) : (GD/∞/∞) model.(15)

Or

(b) (i) Patients arrive at a clinic according to a Poisson distribution at the rate 20 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential, with a mean of 8 minutes. (5)

(1) What is the probability that an arriving patient will not wait?

(2) What is the probability that an arriving patient will find a vacant seat in the room?

(3) What is the expected waiting time until a patient leaves the clinic?

(ii) Discuss (M/E_m/1) : (GD/∞/∞) model. (15)

SECTION B — (10 × 2 = 20 marks)

Answer any TEN questions.

5. Define Travelling salesman problem.

6. What do you mean by Fractional cut?

7. What do you mean by principle of optimality?

8. Define weak maximum in optimization theory.

9. Write the Kuhn-Tucker condition.

10. Define quadratic programming.

11. What do you mean by restricted basis?

12. Define the term Load time.

13. What do you mean by lot-size inventory?

14. Define model with instantaneous model.

15. Identify the stationary policies for the gardener model.

16. Define Priority discipline.

17. Write the three axioms corresponding to an exponential distribution.

18. Write a note about a notation to represent a queue.

19. Write the P-K formula for M/G/1 queue.

———————