(6 pages)

OCTOBER 2012 **P/ID 37456/PMAG**

Time : Three hours Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

Each question carries 2 marks.

1. Define a workless constraints and give an example.

2. Write down the Lagrangian form of d'Alembert's principle.

3. Define ignorable co-ordinates.

4. Define Routhian function.

5. Write down the modified Hamilton-Jacobi equation.

6. State Multiplier rule.

7. Define canonical transformation.

8. Define Poisson bracket.

9. State Einstein's principle of relativity.

10. Define the relativistic Doppler effect.

PART B — (5 × 6 = 30 marks)

Answer ALL questions.

Each question carries 6 marks.

11. (a) Explain holonomic and non-holonomic constriants.

Or

(b) A particle of mass m is suspended by a massless wire of length $r = a + b\cos wt$ $(a > b > 0)$ to form a spherical pendulum. Find the equations of motion.

12. (a) Show that the kinetic energy of a scleronomic system is a homogenous quadratic function of $\dot{q}$'s.

Or

(b) State Kepler's problem and derive the appropriate equations of motion from the Lagrangian.

13. (a) Derive the Hamilton's canonical equations of motion.

Or

(b) Analyse the Kepler problem with regard to Hamilton-Jacobi method.

14. (a) Consider the transformation $Q=\frac{1}{2}\left(q^2+p^2\right)$, $P=-\tan^{-1}\frac{q}{p}$. Show that this transformation is canonical.

Or

(b) Discuss homogeneous canonical transformations.

15. (a) Write a short note on longitudinal contraction.

Or

(b) Discuss the momentum-energy four-vector.

PART C — (5 × 10 = 50 marks)

Answer ALL questions.

Each question carries 10 marks.

16. (a) State and prove Konig's theorem.

Or

(b) Define angular momentum. Find the expressions for the angular momentum of a system of particles with respect to a fixed point with respect to the centre of mass.

17. (a) Find the equations of motion in the form $\ddot{q}_i + f_i(q, \dot{q}, t) = 0$, $i = 1, 2, ..., n$.

Or

(b) Show by means of an example that if a conservative holonomic system does not have a sufficient number of ignorable coordinates, the system will still be separable if it is an orthogonal system.

18. (a) Discuss the brachistochrone problem.

Or

(b) State and prove principle of least action.

19. (a) Consider the transformation

$$Q = q - tp + \frac{1}{2}gt^2 ;\ P = p - gt$$

Find $K - H$ and the generating functions.

Or

(b) Show that

$$[u,v] = -[v,u];\ [u,u] = 0$$

$$[u,[v,w]] + [v,[w,u]] + [w,[u,v]] = 0 .$$

Hence write down the canonical equations of Hamilton interms of Poisson brackets.

20. (a) Discuss the invariant interval, proper time and proper distance.

Or

(b) A particle moves relative to the frame I' with a velocity v' in a direction given by the angle ϕ' measured from the positive x' axis. Find the amplitude and direction of this particle relative to the I frame.
