MCA (Revised)

Term-End Examination

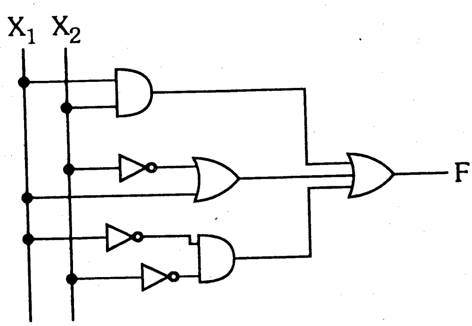
December, 2007

MCS-013 : DISCRETE MATHEMATICS

Time : 2 hours *Maximum Marks : 50*

Note : *Question number 1 is* **compulsory**. *Attempt any* **three** *questions from the rest.*

1. (a) Find the boolean expression for the following circuit. 4



(b) Prove by induction 3

$$1 + 2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1$$

(c) For the sets $A = \{a, b, c, d, e\}$, $B = \{a, b, e, g, h\}$ and $C = \{b, d, e, g, h, k, m, n\}$ prove

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$|X|$ denotes the number of elements in X. 4

(d) By using truth table show that

$(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

is a tautology. 3

(e) In how many ways can a committee of 3 faculty members and 2 students be formed from a group of 7 faculty members and 8 students ? 3

(f) Let $A = \{1, 2, 3, 4, 8\} = B$. R is a relation from A to B. aRb iff a divides b. What are the elements of R ? 3

2. (a) Let A = B = C = R (set of real numbers). Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be defined by $f(a) = a - 1$ and $g(b) = b^2$. Find

(i) $(f \circ g)(2)$

(ii) $(g \circ f)(x)$ 4

(b) Define an equivalence relation. Show that divisibility in the set of real numbers is not an equivalence relation. 4

(c) In how many ways can 6 men and 6 women be seated in a row if men and women must occupy alternate seats ? 2

3. (a) Prove by contrapositive. "Let n be an integer. If n is odd then n^2 is odd." 4

(b) Show that the statements

$(p \wedge q \wedge r') \vee (p \wedge q' \wedge r')$ and $p \wedge r'$

are logically equivalent. 3

(c) Draw the logic diagram of

$(p \wedge q) \vee (q \wedge r')$ 3

4. (a) Write the DNF and CNF for the boolean function f(x, y, z).

x	y	z	f(x, y, z)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

4

(b) Simplify the following boolean expression.

$(a' \wedge b \wedge c) \vee (a' \wedge b' \wedge c) \vee (a \wedge b \wedge c') \vee$
$(a' \wedge b' \wedge c') \vee (a \vee b' \vee c')$ 4

(c) Using truth table show that $p \to q \equiv p' \vee q$. 2

5\. (a) Using Pigeonhole principle show

"If any 14 numbers from 1 to 25 are chosen, then one of them is a multiple of the other." 4

(b) A basket contains 3 apples, 5 bananas, 4 oranges and 6 pears. A piece of fruit is chosen at random from the basket. Compute the probability that

(i) an apple or pear is chosen.

(ii) the fruit chosen is not an orange. 4

(c) Let $R = \{(1, 1), (1, 3), (1, 4)\}$ be a relation on $A = \{1, 2, 3, 4\}$. It is not reflexive. Why ? 2