

MAY 2015

P/ID 17504/PCASD

Time : Three hours

Maximum : 100 marks

PART A — (6 × 5 = 30 marks)

Answer any SIX questions.

1. Construct the truth table for the formula $\neg(P \wedge Q) \iff (\neg P \vee \neg Q)$.
2. Show that $(\neg P \wedge (\neg Q \wedge Q)) \vee (Q \wedge R) \vee (P \wedge R) \iff R$.
3. Show that for any two sets A and B $A - (A \cap B) = A - B$.
4. Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x, y) / x - y \text{ is divisible by } 3\}$ show that R is an equivalence relation. Draw the graph of R .
5. Prove that for any commutative monoid $\langle M, * \rangle$, the set of idempotent elements of M forms a sub monoid.
6. The language $L(G_3) = \{a^n b^n c^n / n \geq 1\}$ is generated by the following grammar $G_3 = \langle \{S, B, C\}, \{a, b, c\}, S, \phi \rangle$ where ϕ consists of the productions $S \rightarrow aSBC$, $S \rightarrow aBC$, $CB \rightarrow BC$, $aB \rightarrow ab$, $bB \rightarrow bb$, $bC \rightarrow bc$, $cC \rightarrow cc$. Find $L(G_3)$.

7. Find a real root of the equation $x^3 - 2x - 5 = 0$ using the bisection method in five stages.
8. Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using Trapezoidal rule with $h = 0.2$.

PART B — (7 × 10 = 70 marks)

Answer any SEVEN questions.

9. Obtain the principal disjunctive and conjunctive normal forms of the following formula
 $(\neg P \rightarrow R) \wedge (Q \iff P)$.
10. (a) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$.
(b) Show that $(x)(P(x) \rightarrow Q(x)) \wedge (Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$.
11. Let $R = \{\langle 1, 2 \rangle, \langle 3, 4 \rangle, \langle 2, 2 \rangle\}$ and $S = \{\langle 4, 2 \rangle, \langle 2, 5 \rangle, \langle 3, 1 \rangle, \langle 1, 3 \rangle\}$. Find $R \circ S, S \circ R, (R \circ S) \circ R, R \circ (R \circ R)$ and $S \circ S$.
12. Let $f(x) = x + 2, g(x) = x - 2$ and $h(x) = 3x$ for $x \in R$, where R is the set of real numbers. Find $g \circ f; f \circ f; g \circ g; f \circ h; h \circ g; h \circ f$ and $(f \circ h) \circ g$.

13. Prove that the order of a subgroup of a finite group divides the order of the group.

14. Explain the following terms :

(a) Phrase structure grammar

(b) Sentential form

(c) Context-free grammar

(d) Regular grammar

with an examples.

15. (a) Find the approximate root of $xe^x = 3$ by Newton's-Raphson method, correct to three decimal places.

(b) Find the root between (2,3) of $x^3 - 2x - 5 = 0$ by the method of false position.

16. Using the Gauss-Jordan method solve the following equations

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$x + y + 5z = 7.$$

17. (a) Using Taylor's series method compute $y(0.1)$ correct to 4 decimal places if $y(x)$ satisfies $y' = x + y$, $y(0) = 1$.
- (b) Solve $\frac{dy}{dx} = 1 - y$ with $y(0) = 0$ at $x = 0.1, 0.2$ using Euler's method.
18. Using Runge-Kutta method of fourth order, solve for y at $x = 0.1, 0.2$ from $\frac{dy}{dx} = y - x$ with $y(0) = 2$.
-