MAY 2014 **P/ID 17504/PCASD**

Time : Three hours Maximum : 100 marks

PART A — (6 × 5 = 30 marks)

Answer any SIX questions.

1. Obtain the principal conjunctive normal form of the formula $(\neg P \to R) \wedge (Q \rightleftarrows P)$. Hence derive the principal disjunctive normal form.

2. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$.

3. For any two sets A and B, prove the following :

   (a) $A - B = A \cap (\sim B)$

   (b) $A \subseteq B \Leftrightarrow \sim B \subseteq \sim A$.

4. For each of the following relations on the set $\{1, 2, 3, 4\}$ decide whether it is reflextive. Whether it is symmetric, whether it is antisymmetric and whether it is transitive.

   (a) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

   (b) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$

   (c) $\{(2,4),(4,2)\}$

   (d) $\{(1,2),(2,3),(3,4)\}$

   (e) $\{(1,1),(2,2),(3,3),(4,4)\}$.

5. Prove that the set S_3 of all permutations of 3 elements form a group under composition.

6. Using bisection method, obtain a root, correct to three decimal places, for the equation $x^3 - 4x - 9 = 0$.

7. Using Gauss elimination method, find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$.

8. Evaluate $I = \int_0^1 \frac{1}{1+x} dx$, correct to three decimal places, using trapezoidal rule. (Use $h = 0.5$)

PART B — (7 × 10 = 70 marks)

Answer any SEVEN questions.

9. Simplify the statement using the terms of logic $(P \vee Q \vee R) \wedge (P \vee T \vee \neg Q) \wedge (P \vee \neg T \vee R)$.

10. Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

11. (a) Let $R = \{\langle 1,2\rangle, \langle 3,4\rangle, \langle 2,2\rangle\}$ and $S = \{\langle 4,2\rangle, \langle 2,5\rangle, \langle 3,1\rangle, \langle 1,3\rangle\}$. Find $R \circ S$ and $(R \circ S) \circ R$.

(b) Let **R** be the set of real numbers. Define $f : \mathbf{R} \to \mathbf{R}$ by $f(x) = x + 4$. Show that f is bijective and find its inverse.

12. Let R denote a relation on the set of ordered pairs of positive integers such that $(x, y)R(u,v)$ iff $xv = yu$. Show that R is an equivalence relation.

13. (a) Show that the set N of natural numbers is a semigroup under the operation $x * y = \max\{x, y\}$. Is it a monoid?

    (b) Write an algorithm that converts an infix expression into reverse polish.

14. Construct a regular grammar which will generate all strings of 0's and 1's having both an odd number of 0's and an odd number of 1's.

15. Using Netwon-Raphson method, obtain a root, correct to three decimal places of the equation $x^4 + x^2 - 80 = 0$.

16. Solve the following system of equations using Gauss-Seidel method.

$$10x + 2y + z = 9$$
$$2x + 20y - 2z = -44$$
$$-2x + 3y + 10z = 22.$$

17. Use the Runge-Kutta fourth order method to find the value of y when $x = 1$ given that $y = 1$ when $x = 0$ and that $\frac{dy}{dx} = \frac{y - x}{y + x}$.

18. Using Taylor series method, find $y(1.1)$ and $y(1.2)$ correct to four decimal places given $\frac{dy}{dx} = xy^{1/3}$ and $y(1) = 1$.
