

DECEMBER 2015

P/ID 17504/PCASD

Time : Three hours

Maximum : 100 marks

PART A — (6 × 5 = 30 marks)

Answer any SIX questions.

1. Define conditional statement, construct truth table for  $(P \rightarrow Q) \wedge (Q \rightarrow P)$ .

2. Show that

$$\left( (P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \right) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R) \text{ is a tautology.}$$

3. Write the properties of binary relations in a set.

4. Let  $X = \{1, 2, 3\}$  and  $f, g, h$  and  $s$  be functions from  $X$  to  $X$  given by

$$f = \{ \langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 1 \rangle \};$$

$$g = \{ \langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 3, 3 \rangle \}$$

$$h = \{ \langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 1 \rangle \};$$

$$s = \{ \langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle \}.$$

Find  $f \circ g$ ;  $g \circ f$ ;  $f \circ h \circ g$ ;  $s \circ g$  and  $g \circ s$ .

5. The language  $L(G_3) = \{a^n b a^m / n, m \geq 1\}$  is generated by the  $G_3 = \langle \{S, A, B, C\}, \{a, b\}, S, \phi \rangle$  where the set of productions is  $S \rightarrow aS$ ;  $S \rightarrow aB$ ;  $B \rightarrow bC$ ;  $C \rightarrow aC$ ;  $C \rightarrow a$ . Find the derivation for the string.
6. Find the real root of the equation  $x^3 - 2x - 5 = 0$  applying the method of bisection. Upto five step.
7. Evaluate  $\int_0^1 \frac{1}{1+x} dx$  correct to three decimal places applying Simpson's 1/3<sup>rd</sup> with  $h = 0.125$ .
8. Use the Runge - Kutta second order method to find the value of  $y(0.1)$  and  $y(0.2)$  when  $x_0 = 0$ ,  $y_0 = 2$  and that  $\frac{dy}{dx} = y - x$ .

PART B — (7 × 10 = 70 marks)

Answer any SEVEN questions.

9. (a) Show that  $R \vee S$  follows logically from the premises  $C \vee D$ ,  $(C \vee D) \rightarrow \neg H$ ,  $\neg H \rightarrow (A \wedge \neg B)$  and  $(A \wedge \neg B) \rightarrow (R \vee S)$ .
- (b) Obtain the principal disjunctive normal form of  $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$ .

10. (a) Show that  $R \rightarrow S$  can be derived from the premises  $P \rightarrow (Q \rightarrow S)$ ,  $\neg R \vee P$ , and  $Q$ .
- (b) Show that  $(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$ .
11. (a) Show that the function  $f \langle x, y \rangle = x + y$  is primitive recursive.
- (b) Show that the function  $f(x) = x/2$  is a partial recursive function.
12. Write down a recursive PL/I program for the factorial function. Give your comments on the procedure.
13. Describe the REVPOL algorithm of converting an infix expression into reverse polish.
14. Solve the following system by Gauss – Seidal method
- $$\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$
15. Find the quadratic factor of the polynomial given by  $f(x) = x^3 - 2x^2 + x - 2$  by using Bairstow's method.

16. Use Romberg's method to compute  $I = \int_0^1 \frac{1}{1+x} dx$  correct to three decimal places with  $h = 0.5, 0.25$  and  $0.125$ .
  17. Use the Runge – Kutta fourth order method to compute  $y(0.2), y(0.4)$  and  $y(0.6)$  given  $y' = 1 + y^2$ ,  $y(0) = 0$  take  $h = 0.2$ .
  18. Obtain explicit predictor – corrector formula using Newton's backward difference interpolation formula.
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