OCTOBER 2013 **P/ID 17504/PCASD**

Time : Three hours Maximum : 100 marks

PART A — (6 × 5 = 30 marks)

Answer any SIX questions.

1. Obtain disjunctive normal forms $\urcorner(P \vee Q) \rightleftarrows (P \wedge Q)$.

2. Show that RVS follows logically from the premises $C \vee D, (C \vee D) \to \urcorner H, \urcorner H \to (A \wedge \urcorner B)$, and $(A \wedge \urcorner B) \to (R \vee S)$.

3. Let R and S be two relations on a set of positive integers $I : R = \{< x, 2x > | x \in I\}$, $S = \{\langle x, \urcorner x \rangle | x \in I\}$ Find $R \circ S, R \circ R, R \circ R \circ R$ and $S \circ R$.

4. Show that the function $f(x, y) = x + y$ is primitive recursive.

5. Prove that a subset $S \neq \phi$ of G is a subgroup of $\langle G, * \rangle$ iff for any pair of elements $a, b \in S, a * b^{-1} \in S$.

6. Find a real roots of the equation $x^3 - x - 1 = 0$ using the bisection method in five stage.

7. Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$. Using the method of false position in five stage.

8. Use Romberg's method to compute $I = \int_0^1 \frac{1}{1+x}\, dx$, correct to three decimal places.

PART B — (7 × 10 = 70 marks)

Answer any SEVEN questions.

9. Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

10. Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

11. (a) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(b) If $A = \{\alpha, \beta\}$ and $B = \{1, 2, 3\}$, what are $A \times B, B \times A, A \times A, B \times B$ and $(A \times B) \cap (B \times A)$?

12. Let $X = \{1, 2, 3\}$ and f, g, h and S be functions from X to X given by

$f = \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 3,1\rangle\}$: $g = \{\langle 1,2\rangle, \langle 2,1\rangle, \langle 3,3\rangle\}$

$h = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,1\rangle\}$; $s = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle\}$

Find $f \circ g; f \circ f; g \circ f; f \circ h \circ g$ and $s \circ s$.

13. Prove that every finite group of order n is isomorphic to a permutation group of degree n.

14. (a) Define pharse structure grammer with an example.

(b) The language $L(G_3) = \{a^n b^n c^n | n \geq 1\}$ is generated by the following grammar

$G_3 = \langle \{S, B, C\}, \{a, b, c\}, S, \phi \rangle$ where ϕ consists of the productions $S \to aSBC, S \to aBC,\ CB \to BC,\ \ aB \to ab$, $bC \to bc, cC \to cc$. Find $L(G_3)$.

15. (a) Find a real root of the equation $f(x) = 2x = \cos x + 3$ by method of iteration.

(b) Use the Newton-Raphson method to find a root of the equation $x = e^{-x}$.

16. Solve the following system by Gauss-Jordan method.

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16.$$

17. Evaluate $\int_0^1 \sqrt{1-x^2}\,dx$, correct to three decimal places by using trapezodal and Simpson's rules to the integral with $h = 10, 20$.

18. Use the predictor-corrector formulae for tabulating a solution of

$10\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ for the range $0.5 \le x \le 1.0$.
