

MAY 2012

P/ID 40001/PPHA

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Time : Three hours

Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

All questions carry equal marks.

1. What are orthogonal functions?
2. State Schwarz inequality.
3. Write down Legendre differential equation and its solution.
4. What is Wronskian? Give an example.
5. State Cauchy's Riemann condition for complex function to be analytic.
6. State Cauchy's integral theorem.
7. What is a Fourier transform?
8. Is Laplace transform operator linear?
9. What is homomorphism?
10. What is a group?

PART B — (5 × 6 = 30 marks)

Answer ALL questions.

All questions carry equal marks.

11. (a) Check if  $\sin x$ ,  $\cos x$  and  $\cos 2x$  are orthogonal functions in the interval  $[-\pi, \pi]$ .

Or

- (b) State and prove Schwarz inequality.

12. (a) Prove that Legendre polynomials are orthogonal to each other.

Or

- (b) Is the point  $x = 0$  a singular point for Hermite differential equation.

13. (a) Find the poles of

(i)  $\frac{1}{z^2 + a^2}$

(ii)  $\frac{1}{(z^2 + a^2)^2}$ .

Or

- (b) Is the function  $(xy - 3y^2 + 4x^2y) + i(x^2 + y^2 - 3x^2y + 3xy^2)$  analytic? Explain.

14. (a) Show that Laplace operator is linear.

Or

- (b) Find the Fourier transform of  $|x|$ .

15. (a) Explain the properties of Abelian group, reducible representation and inverse.

Or

- (b) Construct the character table of  $D_{3h}$  group.

PART C — (5 × 10 = 50 marks)

Answer ALL questions.

All questions carry equal marks.

16. (a) List the properties of linear space.

Or

- (b) Explain Gram-Schmidt process of orthogonalization.

17. (a) How will you find the point  $x = \infty$  is a singular point of linear second order differential equation?

Or

- (b) Show that Legendre polynomials are orthogonal to each other in the interval  $[-1, 1]$ .

18. (a) State and prove Cauchy's Residue theorem.

Or

(b) Derive Cauchy-Riemann conditions for a complex function to be analytic.

19. (a) State the properties of Laplace transform.

Or

(b) Find the Fourier transform of  $f(x) = x^2 + \cos x$ .

20. (a) State and prove great orthogonality theorem.

Or

(b) Discuss about group, class, irreducible representation.