

(6 pages)

MAY 2011

P/ID 40001/PPHA

Time : Three hours

Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

All questions carry equal marks.

1. Prove : If A is an orthogonal matrix and if $B=AP$, where p is non-singular, then PB^{-1} is orthogonal.
2. Prove : If A and B commute and C is orthogonal, then $C'AC$ and $C'BC$ commute.
3. Write down the three possibilities for an analytic function.
4. Evaluate $\int_i^1 (z+1)^2 dz$.
5. Derive Modulation theorem.

6. Prove : If $f(s)$ is Laplace transform of $F(t)$, then that of $e^{at} F(t)$ will be $f(s-a)$.
7. Define Isomorphy.
8. Define Homomorphy.
9. Write a second order linear differential equation.
10. State Strum Liouville theory.

PART B — ($5 \times 6 = 30$ marks)

Answer ALL questions.

All questions carry equal marks.

11. (a) Prove that AA' , where A is n -square, is a diagonal matrix if and only if the rows (or columns) of A are orthogonal.

Or

- (b) Prove : If A is orthogonal and $(I+A)$ is non-singular then $B=(I-A)(I+A)^{-1}$ is skew symmetric.

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12. (a) Prove : If $f(z)$ is analytic and single valued within and on a closed contour and if z_0 is any point within C then,

$$f(z_0) = \frac{1}{2\pi i} \int_c \frac{f(z)}{z-z_0} \cdot dz, \text{ where the integral is}$$

taken in the counter clockwise direction.

Or

- (b) Prove : If $f(z)$ is analytic in a domain D , then its derivatives of all orders exist in D and they are analytic functions in D , then device.

$$f^n(z_0) = \frac{n!}{2\pi i} \int_c \frac{f(z)}{(z-z_0)^{n+1}} \cdot dz. (n=1, 2, 3 \dots).$$

13. (a) Find the Laplace transforms of K_t .

Or

- (b) Find the Laplace transforms of e^{at} .

14. (a) Buildup the point groups.

Or

- (b) Write about point groups and space groups.

15. (a) Derive the orthogonality of eigen function for Legendre polynomials.

Or

- (b) State and derive Sturm-Liouville theory.

PART C — (5 × 10 = 50 marks)

Answer ALL questions.

All questions carry equal marks.

16. (a) Prove : Triangle Inequality.

Or

- (b) Construct orthonormal bases of $V_3(R)$ by gram-Schmidt process, using the given vector

$$[2, -1, 0] ; [4, -1, 0] ; [4, 0, -1] .$$

17. (a) If $f(z)$ is an analytic function of z , P.T.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2.$$

Or

- (b) Apply calculus of residues to prove that

$$\int_0^{2\pi} \frac{\sin^2 \theta}{a + b \cos \theta} \cdot d\theta = \frac{2\pi}{b^2} \left[a - \sqrt{a^2 - b^2} \right]; a > b > 0.$$

18. (a) Find the sine transform of $\frac{e^{-ax}}{x}$.

Or

- (b) Using Laplace transformation method solve, the differential equation $y'' + 9y = 0$; satisfying the initial conditions $y(0) = 0$ and $y'(0) = 2$.

19. (a) Construct a character table C_{2v} .

Or

- (b) Write about successive operation and commutating operation.

20. (a) Solve the Equation $\frac{d}{dx}\left(p \cdot \frac{dy}{dx}\right) + 9y + \lambda y = 0$.

Or

- (b) Derive the equation $\frac{d^2y}{dx^2} + \lambda y = 0$, for a vibrating string damped at $x=0$ and 1.
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