DECEMBER 2015 **P/ID 40001/PPHA**

Time : Three hours Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define vector and vector space.

2. Write the orthonormal set obtained from the orthogonal set of nonzero vectors $\{\vec{v}_{1,}, \vec{v}_{2,....,} \vec{v}_k\}$.

3. In $\sin wx$ and $\cos wx$ are the solutions of the same linear differential equation determine their wronskian. What do you infer from its value?

4. Express the Hermite differential equation in the structure of Liouville form.

5. State the necessary and sufficient conditions for a complex function to be analytic at a point.

6. Distinguish Laurent's series from Taylor's series.

7. Determine the Laplace transform of first derivative of $f(x)$.

8. Obtain the fourier transform of $e^{kt} f(t)$,

9. Define invariant subgroup.

10. What do you mean by representation of a group?

PART B — ($5 \times 6 = 30$ marks)

Answer ALL questions.

11. (a) State the properties of real inner product space.

Or

(b) Verify the Schwarz inequality for the vectors $0.4\vec{i}+1.3\vec{j}-2.2\vec{k}$ and $2\vec{i}+3\vec{j}-5\vec{k}$.

12. (a) How do you identify regular and singular points of a linear differential equation? Write the power series solutions valid near these points.

Or

(b) Express Green's function of $Ly(x)=\delta(x-x')$ in terms of eigen functions of the linear differential operator L.

13. (a) Verify Whether $\overline{Z}$ is an analytic function.

Or

(b) Prove the cauchy's integral theorem for a simply connected region.

14. (a) Find the Laplace transform of $\cos ax$.

Or

(b) Establish the convolution theorem for fourier transform.

15. (a) State and prove Lagrange's theorem.

Or

(b) Explain the concept of homomorphism.

PART C — (5 × 10 = 50 marks)

Answer ALL questions.

16. (a) Verify whether the vectors $4\vec{i}+3\vec{j}-\vec{k}, 2\vec{i}+\vec{j}-3\vec{k}$ and $\vec{i}-4\vec{k}$ are linearly dependent.

Or

(b) Obtain the orthogonal set of vectors from $\vec{a}_1=[1\ 1\ 1\ \ 1]^T$ $\vec{a}_2=[1\ 0\ 0\ 1]^T$ and $\vec{a}_3=[-1\ 0 -2\ \ 1]^T$.

17. (a) State the orthogonality theorem of eigen functions and then using it show that Legendre polynomials are orthogonal.

Or

(b) Obtain the Green's function for the Sturm – Liouville operator.

18. (a) State and prove Cauchy's integral formula.

Or

(b) Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$, where $a > b > 0$.

19. (a) Find the solution of $x''-2x'-8x=0$ $x(0)=3$, $x'(0)=6$ by employing Laplace transform method.

Or

(b) Obtain the Fourier transform of e^{-x^2}.

20. (a) State and prove the orthgonality relation for irreducible representations.

Or

(b) Construct the character table of C_3v.
