

MAY 2013

**P/ID 17451/RCA/
PCAA**

Time : Three hours

Maximum : 75 marks

PART A — ($5 \times 5 = 25$ marks)

Answer ALL questions.

All questions carry equal marks.

1. (a) Find the truth table of $(p \wedge q) \vee (p \wedge r)$.

Or

- (b) Find the number of ways that a party of seven persons can arrange themselves in a row of seven chairs. What if they sit around a circular table?

2. (a) Solve the congruence $7x \equiv 2 \pmod{3}$.

Or

- (b) Define an integral domain and a field. Show that every field is an integral domain.

3. (a) Find the positive root of the equation $x^3 - 2x - 5 = 0$ by iteration method.

Or

(b) A real root of the equation $x^3 - 5x + 1 = 0$ lies in the interval (0, 1). Perform four iterations of the secant method to obtain this root.

4. (a) Solve the following system of equations by Gauss-Jordan method :

$$\begin{aligned}x + y + z &= 9 \\2x - 3y + 4z &= 13 \\3x + 4y + 5z &= 40\end{aligned}$$

Or

(b) Solve the system of equations using the Cholesky method :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}.$$

5. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$.

Or

(b) Using Gaussian three-point formula evaluate $\int_{-1}^1 (3x^2 + 5x^4) dx$. Compare with exact value.

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PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

All questions carry equal marks.

6. (a) Show that the propositions $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent.

(b) Determine the validity of the argument $\sim p \rightarrow q, p \vdash \sim q$.

7. (a) Prove : $\binom{17}{6} = \binom{16}{5} + \binom{16}{6}$.

(b) Find the number of ways that twelve students can be partitioned into three teams containing five, four and three students respectively.

8. (a) Prove using the principle of mathematical induction :

$$1.2 + 2.3 + 3.4 + \dots + n.(n + 1) = \frac{n(n + 1)(n + 2)}{3}.$$

(b) Define a normal subgroup of a group and a group homomorphism . Prove that kernel of a group homomorphism $f : G \rightarrow G'$ is a normal subgroup of G .

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9. Find the positive root of $x - \cos x = 0$ by bisection method.
10. Using Newton-Raphson method, find a real root of $x^3 + 2x^2 + 50x + 7 = 0$.
11. Solve the following system of equations by LU decomposition method :

$$\begin{aligned}4x_1 + x_2 + x_3 &= 4 \\x_1 + 4x_2 - 2x_3 &= 4 \\3x_1 + 2x_2 - 4x_3 &= 6.\end{aligned}$$

12. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using Gaussian method.

13. (a) Using Simpson's rule find the value of $\int_0^2 \frac{dx}{1+x^3}$ dividing the interval $[0, 2]$ into 4 equal parts.
- (b) Evaluate $\int_0^1 \left(1 + \frac{\sin x}{x}\right) dx$ correct to 3 decimal places using Romberg integration.