DECEMBER 2015

P/ID 17451/ RCA/PCAA

Time : Three hours Maximum : 75 marks

PART A — (5 × 5 = 25 marks)

Answer ALL questions.

1. (a) Construct the truth table for $(q \vee r) \rightarrow (p \wedge \neg r)$.

Or

(b) Suppose repetitions are not permitted.

(i) How many three–digit numbers can be formed from the six digits 2,3,4,5,7 and 9?

(ii) How many of these numbers are less than that of 400? How many are even?

2. (a) Prove by Mathematical induction that $2^n > n$ for all $n \in N$.

Or

(b) Let T be the set of all even integers, show that the semigroups $(Z,+)$ and $(T,+)$ are isomorphic.

3. (a) Find an approximate root of $xe^x = 2$ by Regula Falsi method.

Or

(b) Find the positive root of $x^4 - x = 10$. Correct to three decimal places using Newton–Raphson method.

4. (a) Using the Gauss–Jordan method, solve the following equations

$10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 5z = 7$

Or

(b) Solve the following system of equation by Gauss seidel method.

$27x + 6y - z = 85$

$x + y + 54z = 110$

$6x + 15y + 2z = 72$

5. (a) Evaluate $\int_0^5 \frac{dx}{4x+5}$ by Simpson's one third rule and hence find the value of $\log_{e^5}(n = 10)$.

Or

(b) Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using Romberg's method. Hence obtain an approximate value for π.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

6. Establish that

(a) $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Rightarrow (\neg P \vee Q)$.

(b) (i) $\neg(P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$

(ii) $\neg(P \leftrightarrow Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$.

7. Find the elements and the multiplication table of the symmetric group S_3.

8. Using method of false position, find a root of the equation $x^3 - 3x - 5 = 0$.

9. Find the root of $\cos x = xe^x$ by Newton–Raphson method. Take $x_0 = 0.5$.

10. Using Gauss–Elimination method, solve the system.

$3.15x - 1.96y + 3.85z = 12.95$
$2.13x + 5.12y - 2.89z = -8.61$
$5.92x + 3.05y + 2.15z = 6.88$

11. Using Gauss–Seidel method, solve the following system. Start with $x = 1, y = -2, z = 3$

$$x + 3y + 52z = 173.61$$
$$x - 27y + 2z = 71.31$$
$$41x - 2y + 3z = 65.46$$

12. By dividing the range into ten equal parts, evaluate $\int_0^{\pi} \sin x dx$ by Trapezoidal and Simpson's 1/3 rule. Verify your answer with integration.

13. Using Gaussian two point formula, evaluate:

(a) $\int_{-1}^{1} \left(3x^2 + 5x^4\right) dx$

(b) $\int_{0}^{1} \left(3x^2 + 5x^4\right) dx$.
