OCTOBER 2013 **P/ID 17451/ RCA/PCAA**

Time : Three hours Maximum : 75 marks

PART A — ($5 \times 5 = 25$ marks)

Answer ALL questions.

All questions carry equal marks.

1. (a) Show that $(P \vee (Q \wedge R)) \rightleftarrows (P \vee Q) \wedge (P \vee R)$ is a tautology.

Or

(b) Find the number of ways that twelve students can be partitioned into three teams A_1, A_2 and A_3 so that each team contains four students.

2. (a) If $ac \equiv bc \pmod{n}$ and $(c, n) = 1$ then prove that $a \equiv b \pmod{n}$

Or

(b) Let G be the group of nonsingular 2×2 matrices under matrix multiplication. Let H be a subset of G consisting of the lower triangular matrices. Show that H is subgroups of G, but not a normal subgroup.

3. (a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

Or

(b) Perform five iterations of the Muller method to find the root of the equation $\cos x - x e^x = 0$.

4. (a) Using Gaussian elimination find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$.

Or

(b) Solve the system of equations $\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ by the cholesky method.

5. (a) Using Romberg integration, evaluate $\int_0^1 \left(1 + \frac{\sin x}{x}\right) dx$ correct to 3 decimal places.

Or

(b) Using Gauss two point formula, evaluate $\int_{-1}^{1} \frac{1}{1+x^2} dx$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

All questions carry equal marks.

6. (a) Show that $p \wedge q$ logically implies $p \leftrightarrow q$.

(b) Find the total number of positive integers that can be formed from the digits 1, 2, 3 and 4 if no digit is repeated in any one integer.

7. (a) Using mathematical induction show that $3^{2n} - 1$ is a multiple of 8.

(b) Using Euclidean algorithm find the greatest common divisor of 163 and 34.

8. (a) Let $f : G \to G'$ be a homomorphism with kernel K. Prove that K is a normal subgroup of G and the quotient group G/K is isomorphic to the image of f.

(b) Define the ring of polynomials over the field K. If f and g are polynomial in K[t], prove that deg (f g) = deg f + deg g.

9. Using the method of false position find a root of the equation $x^3 - 3x - 5 = 0$.

10. Using secant method determine the root of the equation $\cos x - xe^{x} = 0$.

11. Solve by Gauss-Jordan method :

$$x + y + 3z = 6$$
$$x + 3y + z = 8$$
$$2x + y + z = 5$$

12. Solve the following system of equations by Lu decomposition method.

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$

13. Evaluate the integral $\int_0^1 \frac{dx}{1+x}$ using

(a) Composite trapezoidal rule

(b) Composite Simpson's rule with 2,4, and 8 equal subintervals.