(8 pages)

OCTOBER 2011 **P/ID 17401/RBA**

Time : Three hours Maximum : 75 marks

PART A — (5 × 5 = 25 marks)

Answer ALL questions.

All questions carry equal marks.

1. (a) Construct the truth table for $(p \wedge r) \rightarrow (\neg q \vee p) \rightarrow (q \vee r)$.

Or

(b) Determine the principle disjunctive normal form for the formula $(\neg(P \rightarrow Q)) \rightarrow \neg(Q \wedge \neg R)$.

2. (a) For any two sets A and B, prove that $A - (A \cup B) - A - B$.

Or

(b) Let $x = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(x, \leq)$.

3. (a) Show that the set N of natural numbers is a semigroup under the operation $x * y = \max(x, y)$. Is it a monoid?

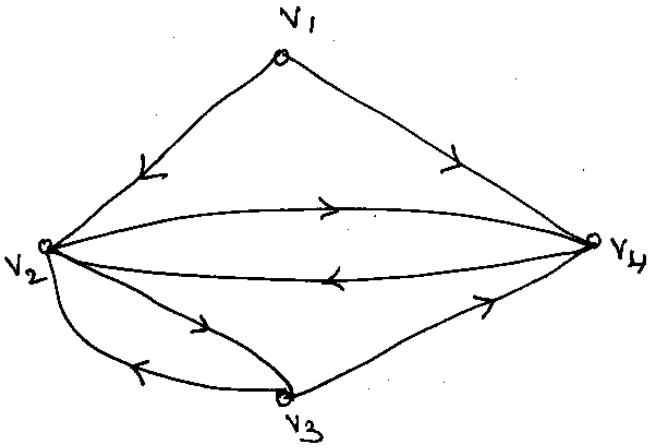
Or

(b) Translate the infix string $(a+b\uparrow c\uparrow d)*(e+f/d))$ to polish.

4. (a) Show that the sum of indegress of all the nodes of a simple digraph is equal to the sum of outdegrees of all its nodes and that this sum is equal to the number of edges of the graph.

Or

(b) Obtain the adjacency matrix of the digraph given below :



5. (a) Let the grammar G be defined by

$S \rightarrow AB$
$A \rightarrow Aa$
$A \rightarrow bB$
$B \rightarrow a$
$B \rightarrow Sb$

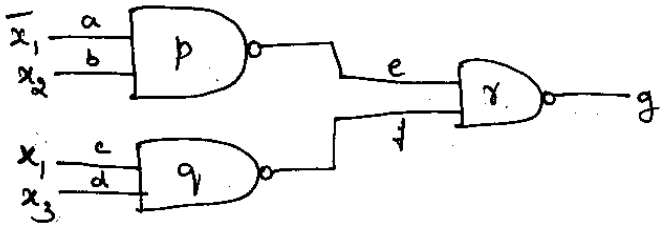
Give derivation trees for the following sentential forms :

(i) baSb

(ii) baabaab.

Or

(b) Construct a table of single faults for the circuit shown below :



PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

All questions carry equal marks.

6. (a) Show that $(P \to Q) \leftrightarrow (\neg Q \to \neg P)$ is a tautology.

(b) Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q.

(c) Show that the following premises are inconsistent ; $E \to S, S \to H, A \to \neg H$ and $E \wedge A$.

7. (a) If the universe of disorder is the set $\{a, b, c\}$, eliminate the quantifiers in the following formulas :

(i) $(x)R(x) \wedge (x)S(x)$

(ii) $(x)R(x) \wedge (\exists x)S(x)$.

(b) Show that

$(x)(P(x) \vee Q(x) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

8. (a) Draw Venn diagrams showing $A \cup B \subset A \cup C$ but $B \not\subseteq C$.

(b) Let $x = \{1,2,3,4\}$ and $R = \{< x, y > / x > y\}$. Draw the graph of R and also give is matrix.

(c) Let $x = \{1,2,.....,7\}$ and $R = \{< x, y > / x - y$ is divisible by $3\}$. Show that R is an equivalence relation.

9. (a) Show that the function $f : N \times N \to N$ is given by $f(x, y) = x + y$ is on to but not one-to-one.

(b) Let R be the set of real numbers. Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = x^2 - 2$ and $g(x) = x + 4$. Find $f \circ g$ and $g \circ f$.

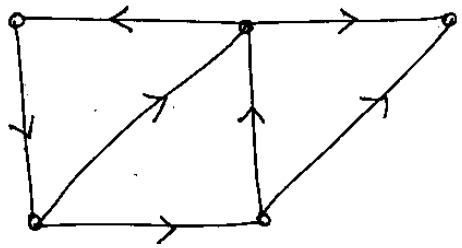
(c) Define a partial recursive function. Show that the function $f(x) = x/2$ is a partial recursive function.

10. (a) Give a context-free grammar which generates $L = \{\omega / \omega$ contains twice as many 0's and 1's$\}$.

(b) Prove that every finite group of order n is isomorphic to a permutation group of degree n.

(c) Let $< G, * >$ and $< H, \Delta >$ be groups and $g : G \to H$ be a homomorphism. Prove that the kernel of g is a normal subgroup.

11. (a) Find the strong, unilateral and weak components of the digraph given below :



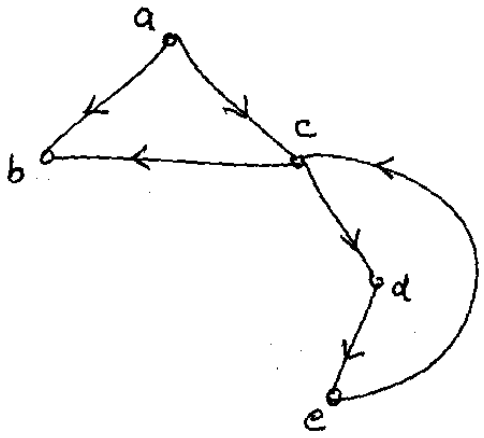
(b) Write an algorithm to obtain a matrix which gives the length of shortest paths between the nodes.

(c) Find the number of different directed trees with three nodes.

12. (a) Prove that a binary tree with n nodes has exactly n +1 null branches.

(b) Formulate an algorithm for the inorder traversal of a binary tree.

(c) Represent the graph given below by a list structure.



13. (a) Let G be a grammar given by

$S \rightarrow yAy$
$A \rightarrow (B$
$A \rightarrow x$
$B \rightarrow Ax)$

Using the parsing algorithm and this grammar G, give a tree of the parse for the strings $y(xx)y$, $(((xx)x)x)y$.

(b) Obtain a test cover for the circuit given below and construct a decision tree for it.