DECEMBER 2014 **P/ID 4521/XDH**

Time : Three hours Maximum :100 marks

SECTION A — (4 × 20 = 80 marks)

Answer ALL questions.

1. (a) (i) Show that if G is simple and $\in > \binom{\gamma - 1}{2}$, then G is connected.

   (ii) Show that $\tau(K_n) = n^{n-2}$.

Or

(b) (i) Show that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

   (ii) Show that a graph G with $\gamma = 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths.

2. (a) (i) State and prove Dirac's theorem on Hamiltonian graphs.

   (ii) Show that the Petersen graph is 4-edge-chromatic.

Or

(b) State and prove Tutte's perfect matching theorem.

3. (a) (i) Show that $\alpha' + \beta' = \nu$, if $\delta > 0$.

(ii) Show that $r(k, k) \geq 2^{k/2}$.

Or

(b) (i) Show that for any positive integer k, there exists a k-chromatic graph containing no triangle.

(ii) Show that if u and v are two vertices of a critical graph G, then $N(u) \not\subseteq N(v)$.

4. (a) (i) Show that B is a bond of a plane graph G if and only if $\{e^* \in E(G^*) | e \in B\}$ is a cycle of G^*.

(ii) Show that each vertex of a disconnected tournament D with $\nu \geq 3$ is contained in a directed k-cycle, $3 \leq k \leq \nu$.

Or

(b) (i) State and prove five colour theorem.

(ii) Show that every tournament is either disconnected or can be transformed into a disconnected tournament by the reorientation of just one arc.

SECTION B — (10 × 2 = 20 marks)

Answer any TEN questions.

5. Define isomorphism of graphs.

6. Define complete k-partite graph.

7. Show that $\delta \leq 2 \in / \gamma \leq \Delta$.

8. Show that G is a forest if and only $\in = \gamma - w$.

9. Show that if G is connected then $\in \geq \gamma - 1$.

10. Show that if G is k-edge connected, then $\in \geq k\gamma / 2$.

11. Define Euler tour.

12. Show that if G is bipartite with bipartition (X, Y) where $|X| \neq |Y|$, then G is Non Hamiltonian.

13. Define 'Hamiltonian-connected'.

14. Find the number of different perfect matchings in $K_{n,n}$.

15. Show that a tree has at most one perfect matching.

16. State Vizing's theorem.

17. Find $\beta(K_{4,6})$.

18. State Brook's theorem.

19. Show that if G is a simple planar graph, then $\delta \leq 5$.

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