MAY 2011 **P/ID 4521/XDH**

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 20 = 80 marks)

Answer ALL the questions.

Each question carries 20 marks.

1. (a) (i) State and prove Sperner's lemma.

(ii) Prove that. any spanning tree $T^* = G[\{e_1, e_2, ...e_{v-1}\}]$ constructed by Kruskal's algorithm is an optimal tree.

Or

(b) (i) If G is a tree, prove that $\varepsilon = v - 1$

(ii) Prove that $\tau(K_n) = n^{n-1}$.

2. (a) (i) If G is a simple graph with $v \geq 3$ and $\delta \geq v/2$, prove that G is hamiltonian.

(ii) State and prove Hall's theorem.

Or

(b) State and prove Tutte's theorem.

3. (a) (i) For any two integers $k \geq 2$ and $l \geq 2$, prove that $r(k,l) \leq r(k,l-1) + r(k-1,\ l)$.

(ii) For any positive integer k, prove that there exists a k-chromatic graph containing no triangle.

Or

(b) (i) State and prove Brooke's theorem.

(ii) If a simple graph G contains no K_{m+1}, prove that G is degree-majorised by Some complete m-partite graph H. Further prove that if G has the same degree sequence as H, then $G \cong H$.

4. (a) (i) State and prove Euler's formula.

(ii) If G is 2-edge-connected, then G has a disconnected orientation.

Or

(b) (i) If D is strict and $\min\{\delta,\ \delta^*\} \geq v/2 > 1$ prove that D contains a directed Hamilton cycle.

(ii) State and prove the five-color theorem.

SECTION B — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

Each question carries 2 marks.

5. Define a spanning subgraph.

6. Define a closed walk.

7. Define a cut vertex.

8. Write Kruskal's algorithm.

9. Define a matching.

10. State Konig's theorem,

11. Define edge chromatic number.

12. State Vizing's theorem.

13. Define covering number.

14. State Schur's theorem.

15. State Dirac's theorem.

16. State Hajos' conjecture.

17. State Kuratowski's theorem.

18. Define a digraph.

19. State max-flow min-cut theorem.