

OCTOBER 2013

P/ID 4521/XDH

Time : Three hours

Maximum : 100 marks

SECTION A — ($4 \times 20 = 80$ marks)

Answer ALL the questions.

Each question carries 20 marks.

1. (a) (i) Show that in any graph, the number of vertices of odd degree is even.
- (ii) Prove that for any graph G , $K \leq K' \leq \delta$.

Or

- (b) (i) Show that a graph is bipartite if and only if it contains no odd cycle.
- (ii) Show that if G is a block with $\gamma \geq 3$, then any two edges of G lie on a common cycle.
2. (a) (i) Show that a non empty connected graph is eulerian if and only if it has no vertices of odd degree.
- (ii) Show that if G is a k -regular bipartite graph with $K > 0$, then G has a perfect matching.

Or

- (b) (i) Show that if G is a simple graph with $\gamma \geq 3$ and $\delta \geq \gamma/2$, then G is Hamiltonian.
- (ii) Let G be a bipartite graph with bipartition (X, Y) . show that G contains a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$.
3. (a) (i) Show that if $\delta > 0$, then $\alpha' + \beta' = \gamma$.
- (ii) Show that if G be a K -critical graph with a 2-vertex cut $\{u, v\}$ then $d(u) + d(v) \geq 3k - 5$.

Or

- (b) (i) Prove that $\gamma(k, l) \leq \binom{k+l-2}{k-1}$.
- (ii) State and prove Brook's theorem.
4. (a) (i) State and prove Euler's formula.
- (ii) Show that for any flow f and any cut $K = (S, \bar{S})$ in N , $\text{val } f \leq \text{cap} K$. Furthermore, equality hold if and only if each arc in (S, \bar{S}) is f -saturated and each arc in (\bar{S}, S) is f -zero.

Or

- (b) Show that if D is strict and $\min \{\delta^-, \delta^+\} \geq \frac{\gamma}{2} - 1$ then D contains a directed Hamilton cycle.

SECTION B — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

5. Find adjacency matrix of the complete bipartite graph $K_{n,n}$.
6. Show that every nontrivial tree has atleast two vertices of degree one.
7. Give an example of a graph G with $K = K'$.
8. Find two distinct Hamilton cycles in K_5 .
9. Define 'K-factor' of a graph.
10. For what values of n does the graph K_n have a perfect matching?
11. Define edge chromatic number and give an example.
12. State Turan's theorem.
13. Define covering number and give an example.
14. Define 'K-Critical' graph G .

15. Define independence number of a graph G.
 16. Find $\chi(K_{m,n})$.
 17. Define 'tournament'.
 18. Draw the Petersen graph and find the chromatic number.
 19. Define plane graph and give an example.
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