

**BACHELOR IN COMPUTER  
APPLICATIONS****Term-End Examination****December, 2007****CS-60 : FOUNDATION COURSE IN  
MATHEMATICS IN COMPUTING**

Time : 3 hours

Maximum Marks : 75

**Note :** Question No. 1 is **compulsory**. Attempt any **two** questions from Questions No. 2 to 5.

1. (a) Find without expanding the value of

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

- (b) Find the value of  $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$ .
- (c) Find the square root of 'i'.
- (d) Prove that  $[f(x) + f(-x)]$  is an even function and that  $[f(x) - f(-x)]$  is an odd function.
- (e) Find  $\frac{dy}{dx}$ , when  $y = \cos(x + y)$ .

- (f) If  $z$  is the product of two complex numbers  $z_1$  and  $z_2$ , prove that

$$|z| = |z_1| |z_2|$$

$$\text{and } \text{Arg } z = \text{Arg } z_1 + \text{Arg } z_2$$

- (g) Prove that

$$\int_{-a}^{+a} f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(x) = f(-x).$$

- (h) Find the derivative of  $\tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right)$  w.r.t.  $\tan^{-1} x$ .

- (i) Find

$$\lim_{x \rightarrow 0} (1+x)^{1/x}$$

- (j) Obtain the equation of the tangent to the circle  $x^2 + y^2 = a^2$  at  $(x_1, y_1)$ .

- (k) If the extremities of a focal chord of the parabola  $y^2 = 4ax$  are  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$ , then prove that  $t_1 t_2 = -1$ .

- (l) Prove that the equation

$$x^2 + 6xy + 9y^2 + 4x + 12y + 5 = 0$$

represents a pair of straight lines.

- (m) Find the equation of the plane passing through the point  $(3, 2, -1)$  and the intersection of the planes  $2x + y + 2z = 9$  and  $4x - 5y - 4z = 1$ .

(n) For any two sets A and B, prove that

(i)  $A \cup A = A$

(ii)  $A \cup B = B \cup A$

(o) For a, b, c being non-zero real numbers, prove that

$$(b + c)(c + a)(a + b) > 8abc$$

$$3 \times 15 = 45$$

2. (a) Find the ranges of x, where the function

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

increases with x and decreases with x.

(b) Find the distance of the point (1, -2, 3) from the plane  $x - y + z = 5$  measured parallel to the line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$$

(c) For any three complex numbers  $z_1, z_2, z_3$ , prove that

$$z_1 \operatorname{Im}(\bar{z}_2 z_3) + z_2 \operatorname{Im}(\bar{z}_3 z_1) + z_3 \operatorname{Im}(\bar{z}_1 z_2) = 0 \quad 5 \times 3 = 15$$

3. (a) Find  $\frac{dy}{dx}$ , when  $y = \tan^{-1} \frac{\cos x + \sin x}{\cos x - \sin x}$ .

(b) Evaluate

$$\int \frac{\tan x \, dx}{\sqrt{a + b \tan^2 x}}$$

(c) Solve by Cardano's method the cubic equation

$$x^3 - 3x + 1 = 0$$

$$4 + 5 + 6 = 15$$

4. (a) If  $I_n = \int_0^{\pi/2} \cos^n x \, dx$ ,

then prove that,

$$I_n = \frac{n-1}{n} I_{n-2}$$

- (b) Obtain the equation of the pair of straight lines passing through the point (2, 3) and perpendicular to the pair of straight lines represented by

$$3x^2 - 8xy + 5y^2 = 0.$$

- (c) Solve the biquadratic equation

$$x^4 - 3x^3 + 3x^2 - 3x + 2 = 0,$$

given that one solution is  $x = i$ .

$$5 \times 3 = 15$$

5. (a) Find the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  in the first quadrant bounded by the co-ordinate axes.

- (b) Find the condition that the two spheres

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

$$\text{and } x^2 + y^2 + z^2 + 2u'x + 2v'y + 2w'z + d' = 0$$

cut each other orthogonally.

- (c) Identify the surfaces represented by the following equations and draw rough sketches of the same :

(i)  $x^2 + y^2 = 16$

(ii)  $y^2 + z^2 = ax$

$$5 + 4 + (3 \times 2) = 15$$