MAY 2011 **P/ID 17451/RCA/PCAA**

Time : Three hours Maximum : 75 marks

PART A — (5 × 5 = 25 marks)

Answer ALL questions.

All questions carry equal marks.

1. (a) Show that the proposition $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction.

Or

(b) English alphabet has 26 letters of which five are vowels. Find the number of five-letter words which contain three different consonants and two different vowels.

2. (a) State and prove the fundamental theorem of arithmetic.

Or

(b) Let Q be the set of rational numbers and let $*$ be the operation on Q defined by $a * b = a + b - ab$. Is $(Q, *)$ a semigroup? Is it commutative? Find the identify element for $*$.

3. (a) Find a root of the equation $x^3 - 4x - 9 = 0$ using the bisection method in four iterations.

Or

(b) Using Bairstow's method obtain the quadratic factors of the following equation (perform two iterations).

$x^4 - 3x^3 + 20x^2 + 44x + 54 = 0$ with $(p,q) = (2,2)$.

4. (a) Solve by Gauss-Jordan method :

$5x - 2y + 3z = 18$

$x + 7y - 3z = -22$

$2x - y + 6z = 22.$

Or

(b) Find the inverse of the matrix :

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}$ by the cholesky method.

5. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Romberg's method. Hence obtain on approximate value for π.

Or

(b) Evaluate $\int_{-2}^{2} e^2\, dx$ by Gauss two-point formula.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

All questions carry equal marks.

6. (a) Show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

(b) Find the truth table for $(p \to q) \vee \sim (p \leftrightarrow \sim q)$.

7. (a) Using Binomial theorem, prove that $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 2^4 = 16$.

(b) Find the number of ways that nine toys can be divided evenly among three children.

8. (a) Using the principle of mathematical induction prove that

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n + (n+1)(2n+1)}{6}.$$

(b) Solve the simultaneous congruence $5x = 2 \pmod 7$ and $x \equiv 2 \pmod 4$.

9. (a) Let G be the group of non-singular 2 × 2 matrices under matrix multiplication. Let K be the subset of G consisting of matrices with determinant 1. Show that K is a normal subgroup of G.

(b) Define an integral domain. Show that the ring Z_{105} of the integers modulo 105 is not an integral domain.

10. Find the positive root of the equation $x^3 - 2x - 5 = 0$ by iteration method.

11. Solve the following system of equations by Gauss elimination method :

$$x + y + z = 6$$
$$3x + 3y + 4z = 20$$
$$2x + y + 3z = 13.$$

12. Solve the following system of equations by the LU decomposition method :

$$4x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 - 2x_3 = 4$$
$$3x_1 + 2x_2 - 4x_2 = 6.$$

13. Evaluate $\int_0^1 \frac{dx}{1+x}$ using :

(a) Trapezoidal rule

(b) Simpson's one third rule

(c) Simpson's three eighth rule.

Take $h = \frac{1}{6}$ for all cases.
