

(7 pages)

MAY 2011

P/ID 17414/RBR

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Time : Three hours

Maximum : 75 marks

PART A — (5 × 5 = 25 marks)

Answer ALL questions.

All questions carry equal marks.

1. (a) An animal feed company must produce 200 kg of a mixture consisting of ingredients  $x_1$  and  $x_2$ . The ingredients  $x_1$  costs Rs. 3 per kg and  $x_2$  costs Rs. 5 per kg. No more than 80 kg of  $x_1$  can be used and atleast 60 kg of  $x_2$  must be used. Formulate the model to minimize the cost of the mixture.

Or

- (b) Solve graphically :

$$\text{Maximize } z = 3x_1 + 2x_2$$

$$\text{Subject to } 2x_1 - x_2 \geq -2$$

$$x_1 + 2x_2 \leq 8$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

2. (a) Construct the dual of the problem

$$\text{Maximize } z = 3x_1 + 10x_2 + 2x_3$$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$

$$3x_1 - 2x_2 + 4x_3 = 3$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Or

- (b) Solve the following transportation problem :

					Supply
3	6	3	1	1	4
2	4	3	2	7	5
1	1	2	1	2	6

Demand 2 2 3 4 4

3. (a) Using Branch-and Bound technique solve the following integer programming problem :

$$\text{Maximize } z = 3x_1 + 3x_2 + 13x_3$$

Subject to the constraints :

$$-3x_1 + 6x_2 + 7x_3 \leq 8$$

$$5x_1 - 3x_2 + 7x_3 \leq 8$$

$$0 \leq x_j \leq 5$$

All  $x_j$  are integers for  $j = 1, 2, 3$ .

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- (b) What are the basic features that characterize a dynamic programming problem?
4. (a) Explain the method of processing  $n$  jobs through  $m$  machines.

Or

- (b) The utility data for a network is given below. Determine the total, free and independent floats, identify the critical path.

Activity : 0-1 1-2 1-3 2-4 2-5 3-4 3-6 4-7 5-7 6-7

Duration : 2 8 10 6 3 3 7 5 2 8

5. (a) What are the different types of stochastic processes? Give a brief account of them.

Or

- (b) At a one-man barber shop, the customers arrive following Poisson process at an average rate of 5 per hour and this is according to exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customer spends in the system.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

All questions carry equal marks.

6. Solve the following linear programming problem by simplex method :

$$\text{Maximize } z = 3x_1 + 4x_2 + x_3 + 7x_4$$

$$\text{Subject to } 8x_1 + 3x_2 + 4x_3 + x_4 \leq 7$$

$$2x_1 + 6x_2 + x_3 + 5x_4 \leq 3$$

$$x_1 + 4x_2 + 5x_3 + 2x_4 \leq 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

7. Solve using Big-M method :

$$\text{Maximize } z = 5x_1 + 8x_2$$

$$\text{Subject to } 3x_1 + 2x_2 \geq 3$$

$$x_1 + 4x_2 \geq 4$$

$$x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

8. Use dual simplex method to solve the following L.P.P :

Minimize  $z = 2x_1 + x_2$

Subject to  $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0$

9. A company has 4 machines on which one has to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table :

Job	Machine			
	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22

Determine the optimum assignment.

10. Find the optimal solution to the following integer programming problem using cutting plane algorithm :

Maximize  $z = x_1 - x_2$

Subject to the constraints :

$x_1 + 2x_2 \leq 4$

$6x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$  and  $x_1$  and  $x_2$  are integers

11. There are five jobs, each of which must go through the two machines A and B in the order AB. Processing times (in hours) are given in the table below :

Job :	1	2	3	4	5
Machine A :	5	1	9	3	10
Machine B :	2	6	7	8	4

Determine a sequence for the five jobs that will minimize the elapsed time T.

12. Cars arrive at a petrol pump, having one petrol unit, in Poisson fashion with an average of 10 cars per hour. The service time is distributed exponentially with a mean of 3 minutes. Find.
- (a) The average number of cars in the system
  - (b) Average number time in the queue
  - (c) Average queue length
  - (d) The probability that the number of cars in the system is 2.
13. A super market has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes, and if the people arrive in a Poisson fashion at the rate of 10 per hour,
- (a) What is the probability of having to wait for service?

- (b) What is the expected percentage of idle time for each girl?
  - (c) If a customer has to wait, what is the expected length of his waiting time?
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