(6 pages)

MAY 2011 **P/ID 17401/RBA**

Time : Three hours Maximum : 75 marks

PART A — ($5 \times 5 = 25$ marks)

Answer ALL questions.

All questions carry equal marks.

1. (a) Show that the formulas $\neg(P \vee Q)$ and $\neg(P \wedge Q)$ are equivalent.

Or

(b) Draw the logic network for the following :

(i) $a \cdot \overline{b} + \overline{a} \cdot b$

(ii) $(a+b) \cdot (\overline{a} + \overline{b})$.

2. (a) If $A = \{\alpha, \beta\}$ and $B = \{1,2,3\}$, find $A \times B$, $B \times A$ and $(A \times B) \cap (B \times A)$.

Or

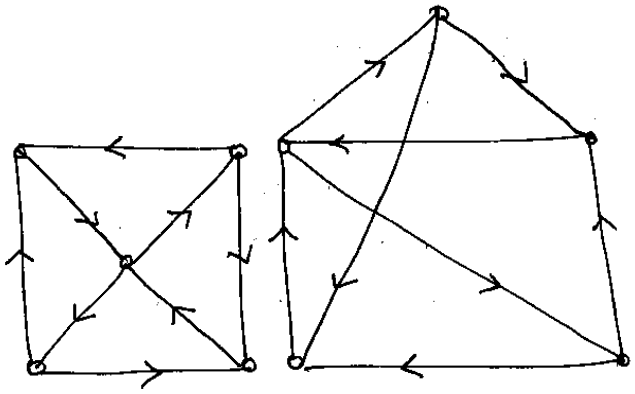
(b) Let $\boldsymbol{R}$ be the set of real numbers and let $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = x^2 - 2$ and $g(x) = x + 4$. Find $f \circ g$ and $g \circ f$. State whether these functions are injective, surjecitive and bijective.

3. (a) Let the alphabet $V = \{a, b\}$ and let A be the set including $\wedge$ of all sequences on $\vee$ beginning with 'a' show that $\langle A, \circ, \wedge \rangle$ is a monoid.

Or

(b) Obtain a grammar which will generate the language $L = \{a^n b a^m / n, m \geq 1\}$. Also derive the sentence $a^2 \ ba^3$ using the grammar.

4. (a) Define isomorphic diagraphs. Show that the two graphs given below are isomorphic.



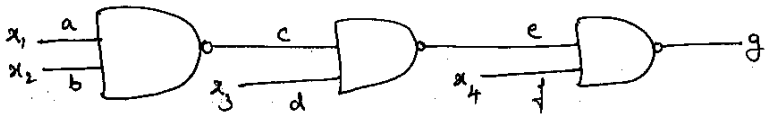
Or

(b) Formulate an algorithm for the inorder traverasal of a binary tree. Explain the algorithm with an example.

5. (a) Let a grammar G be defined by $S \to AB, A \to Aa, \quad B \to a, \quad B \to Sb$. Let baSb be a sentential form. Obtain the pharase, simple phrase and handle for this sentential form.

Or

(b) Generate the fault-detection table for the circuit given below :



PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

All questions carry equal marks.

6. (a) Define a well-formed formula. What are the rules used to generate them?

(b) Construct the truth table to the formula $(P \to Q) \wedge (Q \to R) \to (P \to R)$.

(c) Obtain the principal disjunctive and conjuctive normal forms of $(Q \to P)(\neg P \wedge Q)$.

7. (a) Check whether the following formula is a tautology or a contradiction.
$\neg(P \vee (Q \vee R)) \underset{\leftarrow}{\rightarrow} (P \vee Q) \wedge (P \vee R)$.

(b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q,\ Q \to R, P \to M$ and $\neg M$.

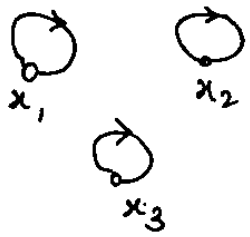
(c) Show that

$(x)(P(x) \to Q(x)) \wedge (x)(Q(x) \to R(x)) \Rightarrow$
$(x)(p(x) \to R(x))$

8. (a) Show that for any two sets A and B, $A - (A \cap B) = A - B$.

(b) Draw Venn diagrams showing $A \cap B = A \cap C$ but $B \neq C$.

(c) Determine the properties of the relation given by the graph below. Also write the corresponding relation matrix.



9. (a) Let R denote a relation on the set of ordered pairs of positive integers such that $\langle x, y\rangle R\langle u, v\rangle$ iff $xv = yu$. Show that R is an equivalence relation.

(b) Let $A = \{a, b, c\}$ and let $P(A)$ be its power set. If $\subseteq$ is the inclusion relation, draw Hasse diagram of $(P(A), \subseteq)$.

(c) Show that $x * y = x - y$ is not a binary operation over the set of natural numbers, but that it is a binary operation on the set of integers. It is commutative or associative?

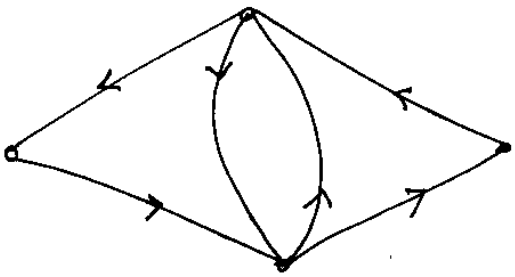
10. (a) Write an algorithm to convert an infix expression into reverse polish expression.

(b) Define a group homomorphism from $\langle G, * \rangle$ to (H, A). Show that kernel of a group homomorphism is a subgroups of G.

11. (a) State and prove Lagrange's theorem.

(b) Prove that every cyclic group of order n is isomorphic to the group $\langle Z_n, t_n \rangle$.

12. (a) Find the adjacency matrix of the diagraph given below :



(b) Give a directed tree representation of the following formula :

$$(P \vee (\neg P \wedge Q)) \wedge ((\neg P \wedge Q) \wedge \neg R)$$

From this representation obtain the corresponding prefix formula.

13. (a) Consider the grammar which has the following productions :

$S \rightarrow aSBc$

$S \rightarrow aBc$

$cB \rightarrow Bc$

$aB \rightarrow ab$

$bB \rightarrow bb$

$bC \rightarrow bc$

$cC \rightarrow cc$

Give derivation tress for the strings abc and $a^2b^2c^2$.

(b) Formulate an algorithm and write a program that will obtain a precedence matrix.

———————