(6 pages)

MAY 2016 **P/ID 17531/PCE13**

Time : Three hours Maximum : 100 marks

PART A — (6 × 5 = 30 marks)

Answer any SIX questions.

1. Solve graphically the following LPP :

Max $Z = 5x_1 + 7x_2$

Subject to the constraints :

$x_1 + x_2 \leq 4,$
$3x_1 + 8x_2 \leq 24,$
$10x_1 + 7x_2 \leq 35,$
$x_1, x_2 \geq 0.$

2. Using VAM method, find an initial basic feasible solution to the following TP.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

3. Explain the Assignment Algorithm.

4. Use dynamic programming to find the value of maximize $Z = y_1 . y_2 . y_3$

Subject to the constraints :

$y_1 + y_2 + y_3 = 5;\ \ y_1, y_2, y_3 \geq 0$.

5. Use graphical method to minimize the time needed to process the following jobs on the machines shown. (i.e.) for each machine find the job which should be done first. Also calculate the total time to compute both the jobs.

Job 1	Sequence time	A	B	C	D	E
		3	4	2	6	2
Job 2	Sequence time	B	C	A	D	E
		5	4	3	2	6

6. Write down the rules for constructing the network.

7. The transition probability matrix of the Markov chain $\{x_n\}$, $n = 1,2,3$ having 3 states 1, 2 and 3 is

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix} \end{array}$$

and the initial distribution is $P^{(0)} = (0.7,\ 0.2,\ 0.1)$. Find $P(X_2 = 3)$.

8. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate :

(a) The mean queue size and

(b) The probability that the queue size exceeds 10.

PART B — (7 × 10 = 70 marks)

Answer any SEVEN questions.

9. Use two-phase simplex method to :

Maximize $Z = \frac{15}{2}x_1 - 3x_2$

Subject to the constraints :

$3x_1 - x_2 - x_3 \geq 3$; $x_1 - x_2 + x_3 \geq 2$; $x_1, x_2, x_3 \geq 0$.

10. Use revised simplex method to solve the LPP :

Maximize $Z = 3x_1 + 5x_2$

Subject to the constraints :

$x_1 \leq 4$; $x_2 \leq 6$; $3x_1 + 2x_2 \leq 18$; $x_1, x_2 \geq 0$.

11. Using the bounded variable technique, solve the following LPP :

Maximize $Z = 3x_1 + x_2 + x_3 + 7x_4$

Subject to the constraints :

$2x_1 + 3x_2 - x_3 + 4x_4 \le 40; -2x_1 + 2x_2 + 5x_3 - x_4 \le 35;$
$x_1 + x_2 - 2x_3 + 3x_4 \le 100$; $x_1 \ge 2$; $x_2 \ge 1$;

$x_3 \ge 3$; $x_4 \ge 4$.

12. Solve the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
b_j	16	18	31	25	

13. Find the integer solution to the LPP :

Maximize $Z = 2x_1 + 2x_2$

Subject to the constraints :

$5x_1 + 3x_2 \le 8$; $x_1 + 2x_2 \le 4$; $x_1, x_2 \ge 0$ and are integers.

14. Find the minimum value of $Z = y_1^2 + y_2^2 + \ldots + y_n^2$ subject to the constraints :

$y_1 . y_2 .... y_n = C$, $y_j \ge 0$ for $j = 1, 2, \ldots n$.

15. A project schedule has the following characteristics :

Activity :	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7
Time :	4	1	1	1	6	5	4	8

Activity :	6-8	7-8	8-10	9-10
Time :	1	2	5	7

(a) Construct PERT network.

(b) Find the critical path and total time duration of the project.

(c) Compute total float for each event.

16. The following table shows the jobs of a network along with their time estimates :

Job :	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
Optimistic (days) :	1	2	2	2	7	5	5	3	8
Most likely (days) :	7	5	14	5	10	5	8	3	17
Pessimistic (days) :	13	14	26	8	19	17	29	9	32

(a) Draw the project network.

(b) Find the expected duration and variance of each activity.

(c) Find the probability that the project is completed in 40 days.

17. A gambler has Rs. 2. He bets Rs. 1 at a time and wins Rs. 1 with probability ½. He stops playing if he loses Rs. 2 or wins Rs. 4.

    (a) What is the t.p.m. of the related Markov chain?

    (b) What is the probability that he has lost his money at the end of 5 plays?

    (c) What is the probability that the games lasts more than 7 plays.

18. At a one-man barber shop, customers arrive according to Poisson distribution with a mean arrival rate of 5 per hour and his hair cutting time was exponentially distributed with an average hair cut taking 10 minutes. It is assumed that because of his excellent reputation, customer were always willing to wait. Calculate the following :

    (a) Average number of customers in the shop.

    (b) Average number of customers waiting for a hair cut.

    (c) The percent of time an arrival can walk right in without having to wait.

    (d) The percentage of customers who have to wait prior to getting into the barber's chair.
