Bachelor in Information Technology (BIT)

Term-End Examination

December, 2007

CSI-32 : DISCRETE MATHEMATICS

Time : 3 Hours *Maximum Marks : 75*

Note : *All questions from Section A are **compulsory**. Attempt any **three** questions from Section B.*

SECTION A

1. State True/False for the following statements and also give reasons for it : *10*

   (i) $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

   (ii) Let N be the set of natural numbers. Then $f : N \to N$ $f(j) = j \pmod 3$ is one-to-one.

   (iii) $A \subseteq A \cup B$ and $A \cap B \subseteq A$.

   (iv) $(Q \to P) \wedge (\neg P \wedge Q)$ is a tautology.

   (v) Product of two even permutations is even.

2. (a) If A = {1, 2, 3} and $A = \left\{\frac{0{\cdot}4}{1}, \frac{0{\cdot}6}{2}, \frac{0{\cdot}3}{4}\right\}$ then find A^c. *3*

   (b) Write in symbolic term the statement and construct truth table.
   The crop will be destroyed if there is a flood. *2*

   (c) Draw a network using *5*

   (i) NOT, AND and OR gates

   (ii) NAND gates only

   for the formula $(P \wedge Q) \vee (\neg R \wedge \neg P)$.

3. (a) Show that $f : X \to Y$ is one-to-one if any proper subset of X is once mapped into proper subset of Y. That is if $A \subset B \subseteq X$, then $f(A) \subset f(B) \subseteq Y$. *5*

   (b) Let F_x be the set of all one-to-one onto mapping from X onto X, where X = {1, 2, 3}. Find all the elements of F_x and find the inverse of each element. *3*

   (c) Write the following into disjunctive normal form : *2*

   $(\neg P \vee Q)$

SECTION B

*Attempt any **three** questions from this section.*

4. (a) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$. *6*

(b) Show that for any two sets A and B

$A - (A \cap B) = A - B$. *4*

(c) Let $f : R \rightarrow R$ and $g : R \rightarrow R$, where R is a set of real numbers. Find fog and gof where $f(x) = x^2 - 2$ and $g(x) = x + 4$. *5*

5. (a) Show that

$f(A \cup B) = f(A) \cup f(B)$

$f(A \cap B) \subseteq f(A) \cap f(B)$

Construct an example to show that in general it is not possible to replace $\subseteq$ by = in the second relation. Under what conditions will $f(A \cap B) = f(A) \cap f(B)$? 9

(b) Let $f(x) = x + 2$, $g(x) = x - 2$ and $h(x) = 3x$ for $x \in R$, where R is the set of real numbers. Find gof, fog, fof, gog, fohog, hof. *6*

6. (a) If relations R and S are reflexive, symmetric and transitive, show that $R \cap S$ is also reflexive, symmetric and transitive. *6*

(b) Prove that $P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)$. *5*

(c) Negate the following statements : *4*

(i) Ottawa is a small town.

(ii) Every city in Canada is clean.

7. (a) Obtain the principal conjuctive normal forms of the following formula : $(\rceil P \rightarrow R) \wedge (Q \leftrightarrow P)$. *6*

(b) Construct the truth table for $\rceil(P \wedge Q) \leftrightarrow (\rceil P \vee \rceil Q)$. *4*

(c) Let A, B, C be three arbitrary sets. Show that *5*

(i) $(A - B) - C = A - (B \cup C)$

(i) $(A - B) - C = (A - C) - B$