OCTOBER 2012 **P/ID 37458/PMAJ**

Time : Three hours Maximum : 100 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

Each question carries 2 marks.

1. Define function of class m.
2. Define tangent surface.
3. Define geoderic curvature.
4. Define rectifying developable.
5. Define Umblic.
6. State Hilbert lemma.
7. Define Christoffel symbols of second kind.
8. Define metric tensor.
9. Define generalized kronecker delta.
10. Define Rieman - Christoffel tensor.

SECTION B — (5 × 6 = 30 marks)

Answer ALL questions.

Each question carries 6 marks.

11. (a) Derive Serret-Frenet formulae.

Or

(b) Show that metric is invariant under a parametric transformation.

12. (a) Prove that the curves of the family $\frac{v^3}{u^2}$ = costant are geoderics on a surface with metric $v^2 du^2 - 2uv\, du\, dv + 2u^2 dv^2 (u > 0, v > 0)$.

Or

(b) State and prove Euler's theorem.

13. (a) If a_{ij} is a tensor, show that A^{ij}, the cofactor of a_{ij} to $|a_{ij}|$, divided by $|a_{ij}| \neq 0$, is a tensor.

Or

(b) Prove that the only compact surfaces of class ≥ 2 for which every point is an umbilic are spheres.

14. (a) Show that every tensor can be written as the sum of a symmetric tensor and a skew symmetric tensor. Is the expression unique? Justify.

Or

(b) Define a relative tensor of weight W. Show that $|g_{ij}|$ is a relative tensor of weight two, with usual notation.

15. (a) State and prove Ricci's theorem.

Or

(b) Prove that Riemann-Christoffel tensor $R_{ijk,l}$ is a skew - symmetric in the first pair of indices as well as in the second pair of indices.

SECTION C — (5 × 10 = 50 marks)

Answer ALL questions.

Each question carries 10 marks.

16. (a) State and prove fundamental existence theorem for space curves.

Or

(b) Calculate the torsion and curvature of the cubic curve $\vec{\gamma} = (u, u^2, u^3)$.

17. (a) State and prove Gauss-Bonnet theorem.

Or

(b) State and prove Minding's theorem.

18. (a) Prove Hilbert's lemma.

Or

(b) State and explain the quotient law of tensors.

19. (a) Show that the Christoffel symbols of second kind are not tensors unless the coordinate transformation is affine.

Or

(b) If $g \left| g_{ij} \right|$, show that $\frac{\partial}{\partial x^i} \log \sqrt{g} = \begin{Bmatrix} \alpha \\ i\,\alpha \end{Bmatrix}$.

20. (a) Show that

(i) $\delta^{ijk}_{ijk} = \lfloor\underline{3}$ if $i, j, k = 1, 2, 3$

Show that

(ii) $\delta^{ij}_{\alpha\beta} = \begin{vmatrix} \delta^i_\alpha & \delta^j_\alpha \\ \delta^\gamma_k & \delta^\gamma_\beta \end{vmatrix}$.

Or

(b) Show that $R^i_{jkl,m} + R^i_{jlm,k} + R^i_{jmk,i} = 0$.