MAY 2015 **P/ID 37458/PMAJ**

Time : Three hours Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define regular function.
2. Define rectifying plane.
3. Write down the canonical geodesic equations.
4. Define Osculating developable.
5. State Hilbert's theorem.
6. Define a contravariant tensor of rank one.
7. Define christoffel symbols of first kind.
8. Define Symmetric and skew symmetric tensors.
9. Define covariant Riemann-Christoffel tensor.
10. Define e-system.

PART B — (5 × 6 = 30 marks)

Answer ALL questions.

11. (a) Find the are length of one complete turn of the circular helix $\vec{r} = (a\cos u, a\sin u, bu), -\infty < u < \infty$, where $a > 0$ and obtain the equations of the helix with s as parameter.

Or

(b) Find the coefficients of the direction which makes an angle $\frac{\pi}{2}$ with the direction whose coefficients are (l, m).

12. (a) Show that the necessary and sufficient condition for a curve to be line of curvature is that $kd\vec{r} + d\vec{N} = 0$.

Or

(b) Prove that every helix on a cylinder is a geodesics.

13. (a) Prove that the only compact surfaces whose Gaussian curvature is positive and mean curvature constant are spheres.

Or

(b) Show that if $A(i,j,k), A^i B^j C_k$ is a scalar for arbitrary vectors A^i, B^j and C_k then, prove that $A(i,j,k)$ is a tensor.

14. (a) If $g = |g_{ij}|$, prove that $\frac{\partial}{\partial x^i} \log \sqrt{g} = \left\{ {\alpha \atop i \ \alpha} \right\}$.

Or

(b) Derive the formulas for covariant differentiation.

15. (a) Prove that the covariant derivative of either of the fundamental tensors is zero.

Or

(b) State the properties of Riemann-Christoffel tensors.

PART C — (5 × 10 = 50 marks)

Answer ALL questions.

16. (a) Derive the characteristic property of a helix.

Or

(b) Define the general surface of revolution and obtain the equation of the anchor ring.

17. (a) State and prove Gauss-Bonnet theorem.

Or

(b) Show that the surface $e^z \cos x = \cos y$ is minimal.

18. (a) State and Prove Hilbert's lemma.

Or

(b) Explain the quotient laws.

19. (a) Obtain the transformation laws for the Christoffel symbols of second kind.

Or

(b) Show that, if $g_{ij} = 0$ for $i \neq j$, then $\left\{ i \, {}^{i}{}_{i} \right\} = \frac{1}{2} \frac{\partial}{\partial x^i} \log g_{ii}$.

20. (a) Establish the identity of Binachi which holds good in the Riemannian space.

Or

(b) Give some applications of the e-systems to determinants.

———————