DECEMBER 2015 **P/ID 37458/PMAJ**

Time : Three hours Maximum : 100 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define an involute.

2. Define r-equivalent representations.

3. State Minding theorem.

4. Define line of curvature.

5. State Hilbert's lemma.

6. Define admissible transformation.

7. Define symmetric tensor.

8. Define Christoffel symbols of first kind.

9. State existence theorem.

10. Define Riemann-Christoffel tensor of the first kind.

SECTION B — (5 × 6 = 30 marks)

Answer ALL questions.

11. (a) Prove that $[\dot{r}, \ddot{r}, \dddot{r}] = 0$ is a necessary and sufficient condition that the curve be plane.

Or

(b) A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

12. (a) Prove that the curves of the family $v^2 / u^2 =$ constant are geodesics on a surface with metric $v^2\, du^2 - 2\,u\,v\,du\,dv + 2\,u^2\, dv^2$ $(u > 0, v > 0)$.

Or

(b) Derive Rodrigue's formula.

13. (a) Prove that the only compact surfaces with constant Gaussian curvature are spheres.

Or

(b) If a transformation of coordinates T possesses an inverse T^{-1} and if J and K are the Jacobians of T and T^{-1}, respectively, prove that $JK = 1$.

14. (a) If a_{ij} is a tensor, show that A^{ij}, the cofactor of a_{ij} in $|a_{ij}|$ divided by $|a_{ij}| \neq 0$, is a tensor.

Or

(b) Explain the fundamental and associated tensors.

15. (a) State and prove Ricci's theorem.

Or

(b) Prove that a necessary and sufficient condition that the metric coefficients $g_{ij}(x)$ reduce to constants h_{ij} in some reference frame Y is that the Christoffel symbols $\left\{ \begin{matrix} k \\ ij \end{matrix} \right\}_y$.

SECTION C — (5 × 10 = 50 marks)

Answer ALL the questions.

16. (a) State and prove the fundamental existence theorem for space curves.

Or

(b) Find the coefficients of the direction which makes an angle $\frac{\pi}{2}$ with the direction whose coefficients are (l, m).

17. (a) State and prove Gauss-Bonnet theorem.

Or

(b) Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

18. (a) State and prove Jacobi's theorem.

Or

(b) (i) If all the components of a tensor vanish in one coordinate system, prove that they necessarily vanish in all other coordinate system.

(ii) Prove that the sum of two tensors which have the same number of covariant and the same number of contravariant indices is again a tensor of the same type and rank as the given tensors.

19. (a) If $g = |g_{ij}|$, prove that $\frac{\partial}{\partial x^k} \log \sqrt{g} = \begin{Bmatrix} \alpha \\ i\alpha \end{Bmatrix}$.

Or

(b) Obtain the transformation laws for the Christoffel symbols of first kind.

20. (a) Show that

$$R_{ijkl} = \frac{1}{2}\left(\frac{\partial^2 g_{il}}{\partial x^j \, \partial x^k} + \frac{\partial^2 g_{jk}}{\partial x^i \, \partial x^l} - \frac{\partial^2 g_{ik}}{\partial x^j \, \partial x^l} - \frac{\partial^2 g_{jl}}{\partial x^i \, \partial x^k} \right).$$

Or

(b) Give some applications of the *e*-systems to determinants.

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