

**BACHELOR IN COMPUTER
APPLICATIONS****Term-End Examination****December, 2007****CS-601 : DIFFERENTIAL AND INTEGRAL
CALCULUS WITH APPLICATIONS**

Time : 2 hours

Maximum Marks : 60

Note : Question number 1 is **compulsory**. Answer any **three** questions from the rest. Use of calculator is **not** allowed.

1. (a) Explain with reasons whether or not the statements given below are true. 3×5=15
- (i) Product of two odd functions is an even function.
- (ii) The domain of definition of $f(x) = \sqrt{4 - x^2}$ is $-2 < x < 2$.
- (iii) If $f'(a)$ exists and is finite then $f(x)$ is continuous at $x = a$.
- (iv) $f(x) = (x - 1)e^x$ is an increasing function for $x > 0$.
- (v) The function $f(x) = \tan x$ has a finite number of vertical asymptotes.

(b) Evaluate :

$$\lim_{\theta \rightarrow \pi/3} \frac{2 \left(\cos \frac{\pi}{3} - \cos \theta \right)}{\sin \left(\theta - \frac{\pi}{3} \right)}$$

4

(c) Evaluate :

$$\int_0^{\pi} \cos^{13} x \, dx$$

3

(d) Prove that

$$\frac{d}{dx} \left[\int_0^{\sin x} \tan \theta \, d\theta \right] = \cos x (\tan \sin x)$$

5

(e) Verify the truth of Rolle's Theorem for the function,

$$y = \cos^2 x \text{ on the interval } \left[-\frac{\pi}{4}, +\frac{\pi}{4} \right].$$

3

2. (a) Evaluate :

$$\int e^x \frac{x^2 + 1}{(x + 1)^2} \, dx$$

4

(b) Using the concept of differential obtain the approximate value of $\sin 61^\circ$, correct upto three places of decimal.

3

(c) Find $\frac{dy}{dx}$, when $y = \tan^{-1} \left(\frac{ax + b}{a - bx} \right)$.

3

3. (a) If $y = e^{a \sin^{-1} x}$, prove that, $(1 - x^2) y_2 - xy_1 = a^2 y$ where y_1 is first derivative and y_2 is second derivative of y . 5

(b) Find the area enclosed between the parabola, $y^2 = 4ax$ and its latus rectum. 5

4. (a) Given $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$,
 $e^4 = 54.60$, evaluate $\int_0^4 e^x dx$ using Simpson's one-third rule taking four equal intervals. 4

(b) Find the curvature of the curve $y = x^2$ at the point (1, 1). 3

(c) If the curves $y = a^x$ and $y = b^x$ intersect at an angle α , then prove that

$$\tan \alpha = \frac{\ln a - \ln b}{1 + \ln a \cdot \ln b} \quad 3$$

5. (a) Prove that,

$$\int_0^1 \frac{\log x dx}{\sqrt{1-x^2}} = \frac{\pi}{2} \ln \frac{1}{2} \quad 5$$

(b) Show that the tangent to the curve,

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

at the point (a, b) is $\frac{x}{a} + \frac{y}{b} = 2$. 5

