MCS-031

MCA (Revised)

Term-End Examination

December, 2007

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest. All algorithms should be written nearer to C/C++ language.*

1. (a) Define different types of asymptotic notations used for representing growth function of algorithms. (Define all the five types).

Solve the recurrence equation given below : *5+5=10*

$f(n) = f(n-1) + f(n-2)$

$f(0) = 0$

$f(1) = 1$

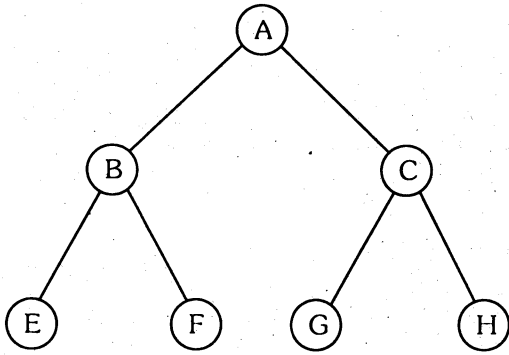
(b) Write the algorithm for HEAP-SORT. Also show functioning of your algorithm on the following array : *6+4=10*

15, 10, 11, 22, 55, 44, 9.

(c) How is dynamic programming different from divide-and-conquer ? Consider the problem of chain matrix multiplication. Devise a solution for the above problem using dynamic programming approach and solve it for the following chain of matrices. *2+8=10*

$$A_{1_{30\times35}} \cdot A_{2_{35\times15}} \cdot A_{3_{15\times5}} \cdot A_{4_{5\times10}} \cdot A_{5_{10\times20}}$$

(d) Write Algorithm for Depth First Search (DFS). Trace how DFS traverses (i.e. discover and visits) the graph given below : *5+5=10*



2. (a) Explain "TURING THESIS". Show that multiplication function is Turing computable i.e. :

$$f(n, m) = n * m$$

is Turing computable. *3+7=10*

(b) (i) Consider the following instance of PCP (Post Correspondence Problem) :

Σ = {a, b}

List L = (ba, abb, bab)

List M = (bab, bb, abb)

Does the above PCP have a solution ? Explain. 5

(ii) Define NP-Complete and NP-haid problems. Write steps through which we can prove that a particular problem is NP-Complete. 5

3. (a) "Greedy Algorithm always gives an optimal solution." Prove or disprove this statement with proper arguments and examples. 5

(b) Find Context Free grammar for the language given below :

$L = \{a^n b^m : n, m \geq 0 \text{ and } n \neq m\}$. 5

(c) Find Finite Automata that accepts strings on input alphabet Σ = {a, b} such that the strings of the language contain total no. of 1's divisible by 3 and total no. of 0's divisible by 2.

eg : 01110, 1001001111, 111, 00 etc. are accepted. 5

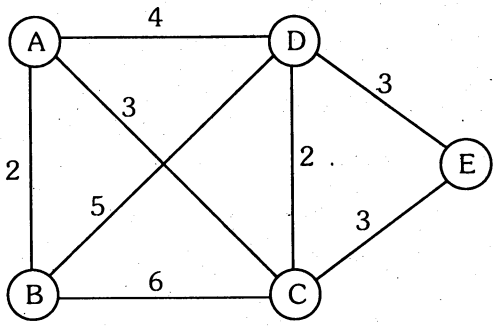
(d) Show that Strassen's matrix multiplication is better than a normal matrix multiplication algorithm. (Show by recurrence relation only, don't write Algorithm). 5

4. (a) Write QUICK-SORT Algorithm and sort the following array showing the steps of Algorithm.

15, 10, 13, 9, 12, 7.

Also find the complexity of this algorithm. *3+4+3=10*

(b) (i) Using Prim's Algorithm, find a minimal spanning tree for the graph given below. 5



(ii) According to CHOMSKY, what are the different types in which grammars are classified ? Explain each with an example. 5

5. (a) Show that clique problem is an NP-complete problem. 6

(b) (i) Explain how Binary Search Algorithm finds an element in an array. Consider the following array :

11, 22, 33, 44, 55, 66, 77, 88

Find 50 and 77 in the above array. Show steps. Compute the complexity of Binary Search. 8

(ii) For $f(x) = 2x^3 + 3x^2 + 1$, show that

(a) $f(x) = 0(x^n)$ for $n \geq 4$

(b) $f(x) \neq 0(x^n)$ for $n \leq 3$ 3+3=6