MAY 2012 **P/ID 37455/PMAF**

Time : Three hours Maximum : 100 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

Each question carries 2 marks.

1. State Liouvile's theorem.

2. State Rouche's theorem.

3. Write the conjugate differential of du.

4. State Mean value theorem.

5. Define the Rieniann 's zeta function.

6. Define a normal family.

7. Define univalent function.

8. Define a periodic function.

9. Define a Sheaf.

10. Define direct analytic continuation of two function elements.

SECTION B — (5 × 6 = 30 marks)

Answer ALL questions.

Each question carries 6 marks.

11. (a) State and prove Weierstrass theorem.

Or

(b) State and prove Schwarz lemma.

12. (a) State and prove Hurwitz theorem.

Or

(b) Evaluate $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}$, $a>1$.

13. (a) For $\sigma>1$, prove that

$$\zeta(s) = -\frac{\Gamma(1-s)}{2\pi i}\int_C \frac{(-z)^{s-1}}{e^z-1}\,dz,$$

where $(-z)^{s-1}$, is defined on the complement of the positive real axis as $e^{(s-1)\log(-z)}$ with $-\pi < \operatorname{Im}\log(-z) < \pi$.

Or

(b) Prove that a locally bounded family of analytic functions has locally bounded derivatives.

14. (a) Prove that the sum of the residues of an elliptic function is zero.

Or

(b) Derive Harnack's inequality.

15. (a) Prove that $\frac{\wp'(z)}{\wp(z)-\wp(u)} = \zeta(z-u)+\zeta(z+u)-2\xi(z)$.

Or

(b) Prove that the homotopy classes of closed curves from z_0, with respect to the region Ω form a group.

SECTION C — (5 × 10 = 50 marks)

Answer ALL the questions.

Each question carries 10 marks.

16. (a) State and prove Cauchy's theorem.

Or

(b) State and prove Residue theorem.

17. (a) Suppose that $u(z)$ is harmonic for $|z|<R$, continuous for $|z|\leq R$. Prove that $u(a)=\frac{1}{2\pi}\int_{|z|=R}\frac{R^2-|a|^2}{|z-a|^2}u(z)d\theta$, for all $|a|<R$.

Or

(b) Obtain the Laurent's expansion of an analytic function in $R_1<|z-a|<R_2$.

18. (a) Prove that the genus and the order of an entire function satisfy the double inequality $h \le \lambda \le h+1$.

Or

(b) Prove that, a family $\Im$ is normal if and only if its closure $\Im$ with respect to the distance function $\rho(f,g) = \sum_{k=1}^{\infty} \delta_k(f,g) 2^{-k}$.

19. (a) Derive the Schwarz-Christoffel formula.

Or

(b) Prove that any two bases of the same module are connected by a uninnodular transformation

20. (a) Prove that every point γ in the upper half plane is equivalent under the congruence subgroup mod 2 to exactly one point in $\overline{\Omega} \cup \Omega'$.

Or

(b) State and prove Monodromy theorem.

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