MAY 2011 **P/ID 4527/XDE**

Time : Three hours Maximum : 100 marks

SECTION A — ($4 \times 20 = 80$ marks)

Answer ALL questions.

Each question carries 20 marks.

1. (a) (i) Prove that the mapping by an analytic function is conformal.

(ii) Prove that a general linear transformation is composed of a translation, an inversion, rotation and homothetic transformations.

Or

(b) (i) Discuss the mapping $\omega = k \cdot \frac{z-a}{z-b}$. List out the properties of Steiner circles determined by a and b.

(ii) Discuss the mapping $\omega = z^{\alpha}$, where α is a real number and positive.

2. (a) (i) Compute $\int\limits_{|z|=2} \frac{dz}{z^2-1}$.

(ii) State and prove Cauchy's theorem for a circular disc.

Or

(b) State and prove Cauchy's theorem for a rectangle.

3. (a) Show that $\frac{\pi}{\sin \pi z} = \lim_{m \to \infty} \sum_{-m}^{m} (-1)^n \cdot \frac{1}{z-n}$.

Or

(b) State and prove the Legendre's duplication formula for the gamma functions $\Gamma(z)$.

4. (a) State and prove Riemann mapping theorem.

Or

(b) (i) Derive the differential equation

$\wp'(z)^2 = 4\wp(z)^3 - g_2\wp(z) - g_3$ under the usual notations.

(ii) Show that the zeros $a_1, a_2, \ldots a_n$ and poles $b_1, b_2, \ldots, b_n$ of an elliptic function satisfy the equation

$a_1 + a_2 + \ldots.. + a_n \equiv b_1 + b_2 + \ldots. + b_n \pmod{M}$.

SECTION B — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

Each question carries 2 marks.

5. Write the Cauchy-Riemann equations of an analytic function $f(z) = u(z) + iv(z)$.

6. If $\omega = f(z(t))$, find $\omega'(t)$.

7. When do you say a mapping is indirectly confirmal?

8. Evaluate $\int_\gamma |dz|$ where γ is the arc $z = z(t) = a + \rho e^{it}, 0 \le t \le 2\pi$.

9. Give the formula for index of a point a, with respect to a closed curve γ.

10. Write the Cauchy's integral formula.

11. Define an essential isolated singularity.

12. When do you say that a cycle γ in Ω is homologus to zero?

13. Find the residue of $\frac{e^z}{(z-a)^2}$ at $z = a$.

14. Show that $\lim_{n\to\infty}\left(1+\frac{z}{n}\right)^n = e^z$.

15. When $|z|<1$, show that $(1+z)(1+z^2)(1+z^4)(1+z^8) = \frac{1}{1-z}$.

16. What is the relation satisfied by the order λ and genus h of an entire function $f(z)$.

17. What is the sum of the residue's of an elliptic function?

18. When do you say ω is a period of a function $f(z)$.

19. Define a meromorphic function.

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