OCTOBER 2012 **P/ID 4527/XDE**

Time : Three hours Maximum : 100 marks

SECTION A — ($4 \times 20 = 80$ marks)

Answer ALL questions.

Each question carries 20 marks.

1. (a) (i) Show that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

(ii) Describe the Riemann surface associated with the function $W = \frac{z}{1-z}$.

Or

(b) (i) Show that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant and the same is true if either the real part, the imaginary part, the modulus or the argument is constant.

(ii) Prove that a general linear transformation is composed of a translation, an inversion, rotation and homothetic transformation.

2. (a) If the function $f(z)$ is analytic on R then show that $\int_{\partial R} f(z)\,dz = 0$.

Or

(b) If $f(z)$ is analytic in an open disk Δ then show that $\int_{\gamma} f(z)\,dx = 0$ for every closed curve γ in Δ.

3. (a) Show that the genus and the order of an entire function satisfy the double inequality $h \leq \lambda \leq h+1$.

Or

(b) State and prove the Legendre's duplication formula for the gamma function $\sqrt{|z|}$.

4. (a) State and prove the Riemann theorem.

Or

(b) State and prove the Schwarz's theorem.

SECTION B — (10 × 2 = 20 marks)

Answer any TEN questions.

5. Define the cross ratio.

6. If $w = (z(t))$ find $w'(t)$.

7. State the symmetry principle.
8. Write the C-R equations of an analytic function.
9. State Cauchy' integral formula.
10. State Liouville's theorem.
11. State Taylors theorem.
12. Write series expansion of arc tan z.
13. Write the expression for $\sin z$.
14. Define equicontinuity.
15. Define the Fourier development.
16. Define unimodular transformation.
17. When do you say that a non constant elliptic function has equally many poles?
18. Define the Weierstrass ρ function.
19. Define harmonic in Ω.