**OCTOBER 2011** **P/ID 4527/XDE**

Time : Three hours Maximum : 100 marks

SECTION A — ($4 \times 20 = 80$ marks)

Answer ALL questions.

Each question carries 20 marks.

1. (a) (i) If $z_1, z_2, z_3, z_4$ are distinct points in the extended plane and $T$ any linear transformation, show that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1, z_2, z_3, z_4)$.

(ii) Find the bilinear transformation which transforms the point 2, 1, 0 into 1, 0, $i$.

Or

(b) (i) Find the bilinear transformation which transforms the half plane $\mathrm{Re}(z) \geq 0$ into the unit circle $|w| \leq 1$.

(ii) State and prove symmetry principle.

2. (a) (i) Show that the line integral $\int_\gamma p\, dx + q\, dy$ defined in $\Omega$ depends only on the end points of $\gamma$ if and only if there exists a function $U(x, y)$ in $\Omega$ with partial derivatives $\frac{\partial U}{\partial x} = p$ and $\frac{\partial U}{\partial y} = q$.

(ii) If the piecewise differentiable closed curve $\gamma$ does not pass through the point $a$, show that the value of $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

Or

(b) By the method of Cantour integration, evaluate $\int_{0}^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$.

3. (a) Show that the genus and the order of an entire function satisfy the double inequality $h \le \lambda \le h+1$.

Or

(b) State and prove Mittag-Leffler theorem.

4. (a) State and prove Riemann Mapping theorem.

Or

(b) (i) Show that the zeros $a_1, a_2, \ldots, a_n$ and poles $b_1, b_2, \ldots, b_n$ of an elliptic function satisfy

$$a_1 + a_2 + \cdots + a_n \equiv b_1 + b_2 + \cdots + b_n \pmod{M}.$$

(ii) Prove that $\begin{vmatrix} \wp(z) & \wp'(z) & 1 \\ \wp(u) & \wp'(u) & 1 \\ \wp(u+z) & \wp'(u+z) & 1 \end{vmatrix} = 0$.

SECTION B — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

Each question carries 2 marks.

5. Define Jordan arc.

6. What is meant by the principal branch of logarithm?

7. When do you say that two points are said to be symmetric with respect to the circle through the points $z_1, z_2, z_3$?

8. Define elliptic transformation.

9. Define rectifiable arc.

10. State Liouville's theorem.

11. State Taylor's theorem.

12. Define pole of a function $f(z)$.

13. State Jensen's formula.

14. Define Riemann Zeta function.

15. When do you say that a family of functions is normal?

16. State Harnack's principal.

17. Define period module of a function $f$.

18. Define Weierstrass $\wp$-function.

19. What is elliptic modular function?

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