DECEMBER 2014 **P/ID 37455/PMAF**

Time : Three hours Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define isolated singularity of a function $f(z)$.

2. When do you say that a cycle is homologous to zero?

3. Define Poisson integral.

4. Give the Taylor series expansion of an analytic function $f(z)$ in the region Ω containing z_0.

5. State Legendre's duplication formula.

6. When do you say that a family of functions is normal?

7. State Schwarz Christoffel formula.

8. State Harnack's inequality.

9. Define sheaf.

10. Define homotopy group of the region Ω at z_0.

PART B — (5 × 6 = 30 marks)

Answer ALL questions.

11. (a) Show that an analytic function comes arbitarily close to any complex value in every neighbourhood of an essential singularity.

Or

(b) State and prove Schwarz lemma.

12. (a) Suppose that $u(z)$ is harmonic for $|z| < R$ continuous for $|z| \le R$. Show that $u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |a|^2}{|z-a|^2} u(z) d\,\theta$ for all $|a| < R$.

Or

(b) If $f(z)$ is analytic in $|z| \le 1$ and $|f| = 1$ on $|z| = 1$, show that $f(z)$ is rational.

13. (a) Show that the necessary and sufficient condition for the absolute convergence of the product $\pi(1 + a_n)$ is the convergence of the series $\sum_1^\infty |a_n|$.

Or

(b) Derive Jensen's formula.

14. (a) Show that a discrete module consists either of zero alone, of the integral multiples $n\ \omega$ of a single complex number $w \neq 0$, or of all linear combinations $n_1 w_1 + n_2 w_2$ with integral coefficient of two numbers w_1, w_2 with non real ratio w_2/w_1.

Or

(b) Show that the zeros $a_1, a_2, \cdots, a_n$ and poles $b_1, b_2, \cdots, b_n$ of an eilliplic function satisfy $a_1 + \cdots + a_n \equiv b_1 + \cdots + b_n (\text{mod } M)$.

15. (a) Show that any elliptic function with periods w_1, w_2 can be expressed in the form.

$$C\ \pi\ \frac{\wp(z) - \wp(a_k)}{\wp(z) - \wp(b_k)}$$, where C is a constant.

Or

(b) Show that

$$\frac{\wp'(z)}{\wp(z) - \wp(u)} = \zeta(z-u) + \zeta(z+u) - 2\zeta(u).$$

PART C — (5 × 10 = 50 marks)

Answer ALL questions.

16. (a) Show that a region Ω is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω an all point a, which do not belong to Ω.

Or

(b) State and prove Rouche's theorem and deduce the fundamental theorem of algebra.

17. Using Residue theory evaluate

(a) $$\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2}$$

Or

(b) $$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}.$$

18. (a) State and prove Wierstrass theorem.

Or

(b) Show that the genus and the order of an entire function satisfy the inequality.

$h \le \lambda \le h+1$.

19. (a) State and prove Riemann mapping theorem.

Or

(b) State and prove Canonical basis theorem.

20. (a) Show that

$$\wp(z+u)+\wp(z)+\wp(u)=\frac{1}{4}\left[\frac{\wp'(z)-\wp'(u)}{\wp(z)-\wp(u)}\right]^2.$$

Or

(b) State and prove Monodromy theorem.
