(6 pages)

OCTOBER 2013 **P/ID 37455/PMAF**

Time : Three hours Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

Each question carries 2 marks.

1. Find the value of $n(\gamma, a)$ if γ lies inside of a circle and a lies outside of the circle.

2. If a and b lie inside of a closed curve γ, find the value $\int_{\gamma}\left(\frac{1}{z-a}-\frac{1}{z-b}\right)dz$.

3. Give the statement of the maximum principle.

4. Find the residue of $\frac{1}{z-a}$ at $z = a$.

5. If $f(z)$ is analytic, find whether $\overline{f(\overline{z})}$ is also analytic or not.

6. Find the value of $\prod_{n=2}^{\infty}\left(1-1/n^2\right)$.

7. What is the value of $\Gamma(3/2)$.

8. When do you say a family functions F is equicontinuous.

9. When do you say the transformation $w_1^1 = aw_2 + bw_1,\ w_2^1 = cw_2 + dw_1$ is unimodular.

10. Define homotopy curves.

PART B — (5 × 6 = 30 marks)

Answer ALL questions.

Each question carries 6 marks.

11. (a) State and prove cauchy's integral formula.

Or

(b) State and prove Morera's theorem.

12. (a) State and prove the Taylor series expansion of $f(z)$ about a point a.

Or

(b) If $u(z)$ is harmonic for $|z| < R$, continuous for $|z| \leq R$, show that

$$u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |a|^2}{|z-a|^2} u(z) d\theta \text{ for all } |a| < R.$$

13. (a) State and prove weierstrass theorem.

Or

(b) Taking $G(z) = \prod_{1}^{\infty} (1 + z/n) e^{-z/n}$ and $H(z) = G(z) e^{\gamma z}$; $\Gamma(z) = \frac{1}{z H(z)}$ prove that $\Gamma(z)\Gamma(1-z) = \pi / \sin \pi z$.

14. (a) State and prove Harnack's principle.

Or

(b) Show that the sum of residues of an elliptic function is zero.

15. (a) Prove that $\wp'(z) = \frac{-\sigma(2z)}{\sigma(z)^4}$.

Or

(b) Prove that $\wp'(z)^2$ is equal to $4\wp(z)^3 - g_2\,\wp(z) - g_3$, under the usual meaning of the notations.

PART C — ($5 \times 10 = 50$ marks)

Answer ALL questions.

Each question carries 10 marks.

16. (a) State and prove that general from of cauchy's theorem.

Or

(b) State and prove Laurent's theorem.

17. (a) Show that $\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$.

Or

(b) If u is continuous at θ_0, show that $P_u(z) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{Re}\frac{e^{i\theta}+z}{e^{i\theta}-z}u(\theta)d\theta$ is hormonic for $|z|<1$ and $\lim_{z \to e^{i\theta_0}} P_u(z) = u(\theta_0)$.

18. (a) Prove that $\frac{\pi}{\sin \pi 2} = \lim_{n \to \infty} \sum_{-m}^{m} \frac{(-)^n}{(z-n)}$

Or

(b) Show that $\pi \cot \pi z = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2z}{z^2 - n^2}$.

19. (a) State and prove Poisson-Jonson's formula.

Or

(b) Show that a family $\mathscr{F}$ is normal if and only if to every compact set $E \subset \Omega$ and for every $\in > 0$, it is possible to find $f_1, f_2 ... f_n \in \mathscr{F}$ such that for every $f \in \mathscr{F}$, $d(f, f_1) < \in$ on E for some f_i.

20. (a) Show that a family $\mathscr{F}$ of analytic functions is normal with respect to C if and only if the functions is $\mathscr{F}$ are uniformly bounded on every compact set.

Or

(b) Discuss the conformal mapping by $\lambda(T)$

taking $w_1 = 1, w_2 = \tau, e_1 = \wp\left(\frac{w_1}{2}\right)$ $e_2 = \wp\left(\frac{w_2}{2}\right)$

and $e_3 = \wp\left(\frac{w_1 + w_2}{2}\right)$.
