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(6 pages)

OCTOBER 2011 **P/ID 37451/PMAA**

Time : Three hours Maximum : 100 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

Each question carries 2 marks.

1. Show that a group of order 25 is abelian.

2. Prove that S_3 is solvable.

3. Find the invariants of the matrix.

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

of a linear transformation T in A(v).

4. Find the companion matrix of the polynomial $(x+1)^2$.

5. If T is unitary and if λ is a characteristic root of T, then show that $|\lambda| = 1$.

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6. Write down a basis of $Q(\sqrt{2})$ over Q.

7. Whether C is a simple extension of **R**? Justify.

8. Find the group of automorphisms of C relative to **R**.

9. Write down the primitive roots of 17.

10. Write down the cyclotomic polynomial $\phi_4(x)$.

SECTION B — ($5 \times 6 = 30$ marks)

Answer ALL questions.

Each questions carries 6 marks.

11. (a) If $0(G)=p^n$, where p is a prime number, then prove that G has a non trivial centre.

Or

(b) Prove that S_4 is solvable.

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12. (a) If V is an n-dimensional vector space over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

Or

(b) Prove that the invariants of a nilpotent linear transformation T are unique.

13. (a) Prove that $T \geq 0$ if and only if $T = AA^*$ for some A.

Or

(b) If $a, b \in K$ are algebraic over F of dimensions m and n, respectively, and if m and n are relatively prime, prove that F(a,b) is of degree mn over F.

14. (a) Show that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f^1(x)$ have a non trivial common factor.

Or

(b) Prove that C is a normal extension of **R**.

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15. (a) If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F, then prove that we can find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

Or

(b) If $a \in H$, the Hurwitz ring of integral quaternions, then show that $a^{-1} \in H$ if and only if N (a) =1.

SECTION C — ($5 \times 10 = 50$ marks)

Answer ALL questions.

Each questions carries 10 marks.

16. (a) State and prove Cauchy's theorem.

Or

(b) Show that any finitely generated module over a Euclidean ring is the direct sum of a finite number of cyclic submodules.

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17. (a) If $T \in A(V)$ has all its characteristic roots in F, then show that there is a basis of V in which the matrix of T is triangular.

Or

(b) Find all possible Jordan forms for all 8×8 matrices having $x^2(x-1)^3$ as minimal polynomial.

18. (a) Show that two real symmetric matrices are congruent if and only if they have the same rank and signature.

Or

(b) Show that $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F.

19. (a) Show that any two splitting fields of the same polynomial over a given field F are isomorphic by an isomorphism leaving every element of F fixed.

Or

(b) Determine the degree of the splitting field of the polynomial $x^6 + x^3 + 1$ over the field of rational numbers F.

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20. (a) Show that a finite division ring is necessarily a communicative field.

Or

(b) Show that every positive integer can be expressed as the sum of squares of four integers.

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