DECEMBER 2015 **P/ID 37471/PMANA**

Time : Three hours Maximum : 100 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the third part of Sylow's theorem.

2. Define finitely generated R-module.

3. Define elementary divisors of a linear transformation.

4. State Jacobson lemma.

5. Define an algebraic number.

6. State the Remainder theorem.

7. Define the Galois group.

8. Define fixed field.

9. Define the adjoint of x.

10. State Frobenius's theorem.

SECTION B — ($5 \times 7 = 35$ marks)

Answer any FIVE questions.

11. Prove that $n(k) = 1 + p + ... + p^{k-1}$.

12. If $u \in V_1$ is such that $uT^{n_1-k} = 0$, where $0 \le k \le n_1$, then prove that $u = u_0T^k$ for some $u_0 \in V_1$

13. If a, b in K are algebraic over F, then prove that $a \pm b$, ab and a/b (if $b \ne 0$) are all algebraic over F.

14. Let $f(x) \in F[x]$ be of degree $n \ge 1$. Prove that there is an extension E of F of degree atmost $n!$ in which $f(x)$ has n roots.

15. Prove that the fixed field of G is a subfield of K.

16. Prove that the general polynomial of degree $n \ge 5$ is not solvable by radicals.

17. If $a \in H$, prove that $a^{-1} \in H$ if and only if $N(a) = 1$.

18. Let C be the set of complex numbers and suppose that the division ring D is algebraic over C. Prove that $D = C$.

SECTION C — (3 × 15 = 45 marks)

Answer any THREE questions.

19. State and prove Cauchy's theorem.

20. Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

21. Prove that the number e is transcendental.

22. If $p(x) \in F[x]$ is solvable by radicals over F, then prove that the Galois group over F of $p(x)$ is a solvable group.

23. State and prove the Left division algorithm.