MAY 2014 **P/ID 37451/PMAA**

Time : Three hours Maximum : 100 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. List all the 2-Sylow subgroups of S_3.

2. Define a cyclic module.

3. If S and T are nilpotent linear transformations which commute, prove that $S+T$ are nilpotent linear transformation.

4. Define the companion matrix of a monic polynomial.

5. If $T \in A(V)$ is Hermition, then prove that all its characteristic roots are real.

6. Find the degree of $\mathbb{C}$ over **R**.

7. State the remainder theorem.

8. Find the splitting field of the polynomial $x^2 + 1$ over the field of rational numbers.

9. If the field F has P^m elements then prove that F is the splitting field of the polynomial $x^{p^m} - x$.

10. If Q is the division ring of real quaternions and if $x \in Q$, define the norm of x.

SECTION B — ($5 \times 6 = 30$ marks)

Answer ALL questions.

11. (a) Show that a group of order 10 is not simple.

Or

(b) State and prove the second part of Sylow's theorem.

12. (a) Show that two nilpotent linear transformation are similar if and only if they have the same invariants.

Or

(b) Prove that the matrix $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent, and find its invariants and Jordan form.

13. (a) If N is a normal matrix, then show that there exists a unitary matrix U such that UNU^{-1} is diagonal.

Or

(b) If L is an algebraic extension of K and if K is an algebraic extension of F, then show that L is an algebraic extension of F.

14. (a) Determine the degree of the splitting field of the polynomial $x^3 - 2$ over the field of rational numbers.

Or

(b) Show that $\mathbb{C}$ is a normal extension of $\mathbb{R}$.

15. (a) If the Galois group of $p(x)$ over F is solvable, then prove that $p(x)$ is solvable by radicals over F.

Or

(b) If the division algebra D over the field of complex numbers $\mathbb{C}$, show that D = $\mathbb{C}$

SECTION C — (5 × 10 = 50 marks)

Answer ALL questions.

16. (a) If P is a prime numbers and $P^m | o(G)$, then show that G has subgroup of order P^m.

Or

(b) Show that every abelian group is the direct product of cyclic groups.

17. (a) If all the distinct characteristic roots $\lambda_1 ...., \lambda_k$ of T lie in F, then prove that V can be written as $V = V_1 \oplus ..... \oplus V_k$. Where $V_i = \{v \in V | v(T - \lambda_i)^{l_i} = 0\}$ and T_i has only are characteristic root, λi, on Vi.

Or

(b) Show that sand T in $A_F(V)$ are similar if and only if they have the same elementary divisions.

18. (a) State and prove sylvester's law.

Or

(b) If L is a finite extension of K and if K is a finite extension of F, then show that L is a finite extension of F and also prove that $[L:F]=[L:K]\ [K:F]$.

19. (a) Show that the existence and uniqueness of the splitting field.

Or

(b) Show that any finite extension of a field of characteristic 0 is a simple extension.

20. (a) Show that the multiplicative group of nonzero elements of a finite filed is cyclic.

Or

(b) State and prove wedderburn's theorem on finite division rings.

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